RAILWAY CURVES - SHUNK

A

PRACTICAL TREATISE

ON

RAILWAY CURVES

AND

LOCATION,

FOR YOUNG ENGINEERS.

CONTAINING A FULL DESCRIPTION OF THE INSTRUMENTS, THE MANNER OF ADJUSTING THEM, AND THE METHODS OF PROCEEDING IN THE FIELD,—NEW AND SIMPLE FORMULÆ FOR COMPOUND AND REVERSE CURVING,—RULES FOR CALCULATING EXCAVATION AND EMBANKMENT,—STAKING OUT WORK, &C., TOGETHER WITH TABLES OF NATURAL SINES AND TANGENTS, RADII, CHORDS, ORDINATES, AND OTHERS OF GENERAL USE IN THE PROFESSION.

BY

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PREFACE.

The located line for railway is a series of curves and straight lines, or tangents. These are first plotted to a large scale from data gathered on preliminary survey. It is therefore desirable that all explorations should be made with extreme care, as upon their correctness depend, in no small degree, the labour and time required in location. It were better for accuracy that all angles should be made and recorded from the plates, and the needle used only as a test, or check. Good chaining is indispensable. Great attention, too, should be given to the proper use of the slope instrument. By these means a working map can be constructed in the office upon which the proposed location, grade lines, &c., may be traced with tolerable resemblance to fact. Still many errors attach to both data and map, and these, together with the unexpected obstacles encountered in the field, require ready knowledge of the means for overcoming them.

It has been my design to present this knowledge to my younger fellows in the profession. I have endeavoured to

do it lucidly and concisely—without supposing unusual cases—without prolix proof or complex figuring. The problems given are of frequent occurrence, and the tables appended will be found useful and correct.

To STRICKLAND KNEASS, a gentleman whose professional abilities are well known, I return thanks for valuable assistance. I would likewise make my acknowledgments, for useful suggestions, to CHARLES DELISLE, an engineer of high mathematical attainment.

I am aware that much more might have been said—much more suggested—on the subject of location; but a field book being the object, the compact plan precluded any extensive essay.

If the work with brevity combines clearness, and is comprehensive withal, it is the work intended.

W. F. SHUNK.

CONTENTS.

TABLES.

EXPLANATIONS.

All railway curves are parts of circles. They are designated *generally* from their character as simple, compound, or reverse; and *specifically* from the central angle subtended by a chord of 100 feet at the circumference, this being the length of the chain in common use. It is found that the circle described with radius of 5730 feet has a circumference of 36,000 feet. Since there are 360° in the circle, the central angle subtended by a chord of 100 feet is, in this case, equal to 1°, and the curve is named a *one degree* curve. So likewise in a circle with radius of 2865 feet, half of 5730, the central angle corresponding to the chord 100 is 2°; the curve is then called a *two degree* curve.

The beginning of a curve is called the point of curvature, or simply the P. C., and its termination the point of tangent, marked P. T.

A compound curve is composed of two curves of different radii, turning in the same direction, and having a common tangent at their point of meeting. This point is called the point of compound curvature, or P. C. C.

A reverse curve is composed of two curves turning in different directions, and having a common tangent at their point of meeting, which latter is named the point of reversed curvature, or the P. R. C.

All sines and tangents made use of in this work are from the table at the end of the volume. For calculating curves it is not necessary to use more than four decimals.

A Bench is a shoulder hewn with the axe on the buttressed base of a tree, and so shaped at the top as to afford footing to the rod. The tree is blazed and the elevation of the bench marked on it with red chalk. Benches serve as permanent reference points to the level. They are placed, where it is possible, about one thousand feet apart.

Points. The operation termed pointing is the fact of putting a peg firmly into the ground, and of driving in its top a tack, or making thereon an indentation whose place is indicated by cross keel marks, directly in the line of collimation of the transit. Thus true lines are traced on the ground, and angles measured accurately. When the transit is set over a point it is so posited that the plumb hangs immediately above the tack head. If the head plate of the tripod be much inclined the plumb should be examined after levelling the instrument, as that operation disturbs it to some extent.

Stations. The line of a survey is marked on the ground at regular intervals, by stakes two feet in length, blazed, and numbered from 0 up in arithmetical progression. These stakes are named stations. On exploration they are commonly placed two hundred, and on location, one hundred feet apart.

It is customary, when locating, to drive pegs even with the surface along the true line, and to place the stations a couple of feet to the right, numbers facing in, to show their

position. The pegs are less liable to disturbance from frost, animals, &c.

In locating for construction stakes are driven on sharp curves at intervals of 50, sometimes 25 feet.

The Chain in general use for railway surveys is made of soft iron. It is 100 feet long, and divided into 100 links, each one foot in length. At every tenth link is attached a brass drop, toothed so as to indicate its distance from the end. It presents the advantages of durability, accuracy, and expedition.

A PRACTICAL TREATISE

ON

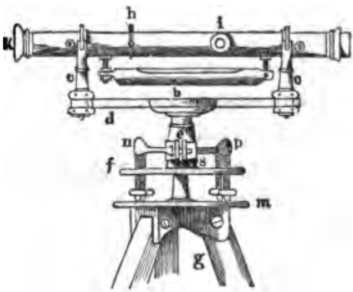
RAILWAY CURVES AND LOCATION.

ARTICLE I.

OF THE INSTRUMENTS.

THE LEVEL.

The level is an instrument used in ascertaining the undulations of the ground along the line of a survey, and of measuring these irregularities accurately in reference to an assumed base called the datum. It consists mainly of the telescope *k i*, the spirit-level and its encasement *b*, the Y's *c c*, the rectangular bar *o d*, the axis *e*, the plates and levelling screws *f m*, and the tripod *g*.



In the tube of the telescope at *h*, and at right angles to its axis, is placed a flat ring, called the diaphragm. To this ring the cross-hairs are attached—two delicate spider lines stretched over it vertically and horizontally, and intersecting at the centre. It is held in position by four

slightly movable screws, which pierce the tube in the direction of the "cross-hairs." *i* is a milled head for adjusting the focus of the object glass, and *k* an inserted tube, containing several lenses, which may be moved out or in so as to make the spider-lines distinctly visible.

A straight line looked along from the eye glass at *k* through the intersection of the cross-hairs is the *line of sight*, technically named the *line of collimation.*

The immediate supports of the telescope are called the Y's, from their resemblance to that letter. If a small arch were sprung between the two legs of the Y it would give a good idea of the clasping pieces which hold the telescope in place. They are jointed to one leg and secured to the other by pins which may be withdrawn and the pieces turned back in order to remove the telescope, or change it end for end.

The Y's are attached at right angles to the bar *d*, which, again, is connected firmly at right angles with the hollow axis *e*. This latter fits closely over and is revolvable horizontally around a *solid* axis *s*, which, passing through the plate *f*, is secured to the head of the tripod by means of a loose ball-and-socket joint. The plate *f* has four levelling screws inserted in it; with these the instrument may be brought to a horizontal position even when the lower plate is considerably inclined.

One of the Y's is movable for a short space up or down by means of the capstan-head screw *o*. The spirit-level is likewise movable both vertically and laterally by means of screws at either end.

n is a clamp screw, and *p* a tangent-screw for slightly turning the telescope in a horizontal direction.

To adjust the Level.

First. To make the line of collimation coincide with the axis of the telescope.

Set the instrument firmly, and direct the telescope toward some distant, distinct object, such as a nail-head. Clamp fast, and with tangent-screw fix the line of collimation upon the object accurately. Revolve the telescope half way round in the Y's, *i. e.* until the bubble is above it, and if the horizontal spider-line still covers the point, it requires no adjustment. If it does not, reduce the error one-half by means of the diaphragm screws, and complete the reduction with the capstan-head screw. Revolve the telescope round to its first position, and if the horizontal line and point do not then coincide, repeat the operation until they do, in any position of the telescope. In similar wise the vertical hair may be adjusted, when the line of collimation should cover the point through an entire revolution of the telescope.

Great care should be taken in this as well as in all other adjustments of cross-hairs, that the opposite screw of the diaphragm be loosened before tightening its fellow, or injury to the instrument must result.

Second. To make the axis of the spirit-level parallel to the line of collimation.

With levelling screws bring the bubble to the middle of its tube, reverse the telescope in its Y's, and if the bubble does not then stand in the middle correct one-half the deviation with the screw at the left end of the bubble-case, and the other half with the capstan-head screw. Again reverse the telescope in its Y's, and, if necessary, repeat the operation.

Now revolve the telescope a short distance in its Y's, so as to bring the spirit-level to one side of its lowest position. If the bubble deviates from the middle, correct the error

with the lateral screws at the right end of the bubble-case, and examine the previous adjustment before lifting the instrument.

Third. To bring the line of collimation parallel to the bar.

Turn the telescope until it stands directly over two of the levelling screws, and with them bring the bubble to the middle of the tube. Then revolve the telescope horizontally until it stands over the same screws, changed end for end. If the bubble does not still stand in the middle of the tube, correct one-half the deviation with the capstan-head, and one-half with the levelling screws.

Place the telescope over the other levelling screws and proceed in a similar manner, and continue the corrections until the bubble stands without varying during an entire revolution of the instrument upon its axis.

This completes the adjustment of the level.

THE ROD.

The rod used in levelling consists of a staff and a target, which latter is so attached to the staff as to be movable along it from end to end. The rod is commonly seven feet long, but, being composed of two rectangular pieces fitted together by means of a sliding groove, it can be extended to nearly double that length. It is graduated to feet and tenths of a foot. The target is a circle of wood or iron, usually four-tenths in diameter, and divided into quadrantal sectors by a horizontal and vertical line which intersect at its centre. The sectors are painted alternately red and white, so that their dividing lines are visible at a considerable distance. On the back of the target, where it meets the graduated side of the rod, is fixed a chamfered brass edging, whereon the space of one-tenth is graven from the centre down. This is subdivided into ten spaces marking

hundredths, and these latter divided into halves, so that the height of the middle of the target above the base of the rod may be accurately read to within ·005 of a foot.

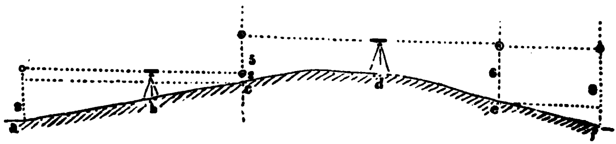
There is a similar graduated tenth on the standing part of the rod, to be used for high sights when the sliding groove comes into play.

Both target and rod are provided with clamp screws.

LEVELLING.

The operation technically called levelling is performed thus:—

Suppose *a* the starting point, or zero, in reference to which all the inequalities of the surface along the line of survey are measured, as at the points *c*, *e*, *f*. The horizontal line *a f* is called the *datum* line. This is arbitrarily



assumed. It may be considered, for example, at any dis tance *above* the point *a*, and the irregularities of the ground measured from an imaginary level line in ether; but for convenience of figuring, and other politic reasons, it is customary in seaport towns to take high tide as datum. Inland, the summer surface of the nearest stream, or, when commencing on a ridge, the highest neighbouring knoll is assumed.

Well! suppose *a* to be zero, and the instrument, for instance, set and levelled at *b*. Stand the rod at *a*, and slide the target up until its cross-lines are covered by the cross-hairs in the telescope; *i. e.*, until the line of collimation coincides with the centre of the target. The leveller directs the movements of the target by raising or lowering

his hand. A circular motion of the hand signifies "make fast." The bubble should always be examined before the rod is taken down, and the latter should be read twice, or, if convenient, shown to the leveller, in order to guard against mistake. If in this case it reads 8 feet, the height of the instrument is then 8 feet above *a*. To find the elevation of *c* above *a*, take the rod thither and lower the target until coincidence results as before. If the rod reads 2 feet, of course *c* is 8 — 2 = 6 feet above *a*.

If it is necessary to lift the instrument here, a small peg is driven at *c* before sighting to that point, to insure firm footing for the rod. Sighting back from the new position, *d*, the rod reads 5 feet; then 5 + 6, the elevation of *c* above *a*, = 11, the height of the telescope at *d* above *a*. If at *e* the reading is 6, the elevation of that point is 11 — 6 = 5, and if at *f* the reading is 8 the elevation of that point in reference to *a* is 11 — 8 = 3, marked + 3.

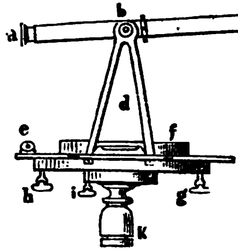
The rule, therefore, in levelling is, at each new stand of the instrument, to add the reading of the rod sighted back at, to the discovered elevation of the point at which the rod stands, for the height of the instrument; and to *subtract* from this height the reading of the rod at any points observed from the new position in order to find the elevation of those points. The above is noted in the field-bock as follows:

Station.	Rod.	Height of Instrument.	Total, or Elevation.
a	8·00	8·00	00
c	2·00		+ 6.00
	5·00	11·00	
e	6·00		+ 5·00
f	8·00		+ 3·00

The advantages of this method of levelling over the old system of backsights and foresights are, that it affords readier facilities for testing the correctness of the work, and it may be carried on more rapidly. By the old plan each sight at the rod was linked with that which preceded it, and added one more to a continuous calculation in which a single error affected all the following work. Here, however, if haste is required, the calculation of the intermediate sights or "cuttings" may be omitted entirely while in the field, the reading of the rod only being set down; the "totals" may be worked from peg to peg, and the liabilities to mistake thus decreased about eighty per cent.

OF THE TRANSIT.

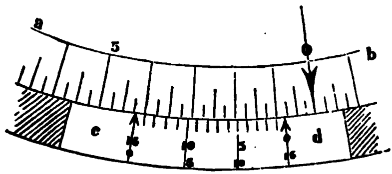
The transit is an instrument for measuring horizontal angles. It consists of the telescope *a c*, the Y's *d*, the compass-box, &c., *e g*, and the axis *k*. The telescope is furnished like that of the level, and the instrument is similarly fitted to its tripod. The telescope revolves in a vertical circle, and is attached to the Y's by means of a transverse axis whose extremities turn in smooth journals at the head of the Y's. The body of the instrument at *f* contains a magnetic needle, with its usual circular surrounding, graduated to degrees and quarter degrees. The flooring of this box has, on one side, an opening with chamfered edge upon which the vernier is engraved. This latter, together with the telescope, Y's, and all the upper part of the instrument, is made to revolve by means of the screw *h*, upon a solid plate beneath, which is likewise graduated from 0 to 180° each way. Thus angles may be measured



accurately without using the needle at all. It need be regarded merely as a check. *g* is a clamp screw for securing the plates together, and *i* a screw for fastening the needle so as to prevent its vibrations while the instrument is being carried from place to place. A plumb is suspended from the axis of the transit, by means of which its centre may be placed over a point on the ground.

THE VERNIER.

The vernier, in the transit, is a graduated index which serves to subdivide the divisions of the graduated arc on the lower plate. There are many varieties of the vernier, but familiarity with one renders easy the acquaintance with all, since the same general principle is pervading.



The figure represents a common form. Let *a b* be part of any graduated arc, and *c d* the vernier. It will be observed that the degrees on the limb are divided into spaces of 15′ each. Now if the vernier be made equal in length to fourteen of those spaces, and be further divided into *fifteen equal parts*, it is evident that each of these parts will contain 14′.

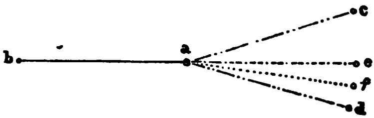
Then, if 0 of the vernier coincides with any division of the limb, the first line of the vernier to the left will be just one minute behind the first line of the limb to the left; the *second* vernier line *two* minutes behind the second limb line, and so on; so that if the vernier be moved to the left over the space of 15′ on the limb, the lines from 0 15 of the vernier would coincide successively with lines

of the limb, and thus any angle may be read accurately to minutes.

The vernier in the figure reads 48′ to the left. A vernier graduated decimally is much more convenient on railway locations than those with the common graduation to minutes. This is principally on account of its adaptedness to running in curves when the 100 feet chain is used. The work can be done with more ease and rapidity. One objection to it is that the tables in general use are calculated for degrees and minutes.

TO ADJUST THE TRANSIT.

Place the instrument firmly at *a*, level it, clamp all fast, and with tangent-screw set the cross-hairs on the point *b*, at any convenient distance. Reverse the telescope on its axis, and fix another point in the opposite direction,



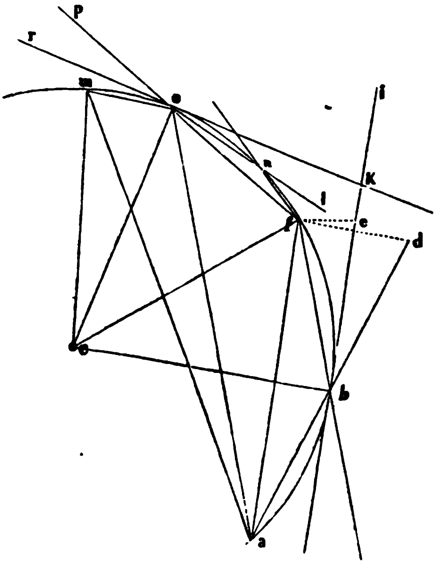
as nearly as possible equidistant from *a*. Now loose the lower clamp and revolve the entire upper part of the instrument half way round on its axis. Clamp fast, and having brought the cross-hairs again to coincide with *b*, reverse the telescope. If the sight strikes as before, the instrument is in adjustment. If not, place another point, *d*, where it does strike, and suppose *c* to be the point previously fixed: the point *e*, midway between *d* and *c*, is then in the straight line. With the adjusting pin carefully place the vertical cross-hair upon *f*, distant from *d* one-quarter of the space *d c*—with tangent-screw set it on *e*, and reverse the telescope. If the points have been correctly placed, and the hair properly moved, the sight will strike *b*, and the adjustment is complete.

After finishing this adjustment, the telescope may still not revolve truly in the meridian. This inaccuracy there is no method of removing in the field. It should be sent to an instrument-maker for repairs.

ARTICLE II.

PRELIMINARY PROPOSITIONS.

1. *In any circle* the angle $o\,c\,f$ at the centre, subtended by the chord $o\,f$, is double the angle $o\,a\,f$, at any part of the circumference on the same side of the chord.



2 *The angle $f\,b\,e$, formed* by any chord $f\,b$, with a tangent at either extremity, is called a *tangential* angle.

and is equal to half the angle *f c b* at the centre, or is equal to the angle *f a b* at the circumference.

3. *The exterior angle d b f*, formed at the circumference by the two equal chords *a b*, *b f*, is called a *deflexion* angle, and is equal to the central angle *b c f*, or double the tangential angle *e b f*. *d f* is called the *deflexion distance*, and *e f* the *tangential distance.*

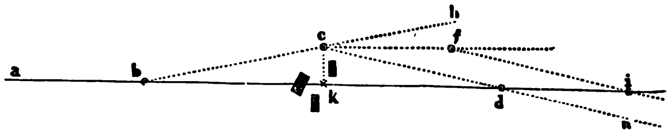
4. *The exterior angle p o m* of two unequal chords, is equal to the sum of their tangential angles, or half the sum of their central angles.

5. *The exterior angle i k o*, formed by tangents, is equal to the central angle *b c o*, subtended by the chord which connects their points of contact with the curve.

ARTICLE III.

TO AVOID AN OBSTACLE IN THE LINE OF TANGENT.

A GLANCE at the figure will show, that having deflected to *c*, and placed the instrument at that point, the angle *h c d* must be made equal to twice *d b c*, and the distance



c d equal to the distance *b c*. Still another deflection at *d*, equal to the original angle turned, is necessary in order to sight again along the tangent.

Should the obstruction be continuous a parallel line may be run, as from *c* to *f*, by deflecting at *c* an angle

equal to that at *b*, and at *f*, repeating the deflection in order to strike tangent.

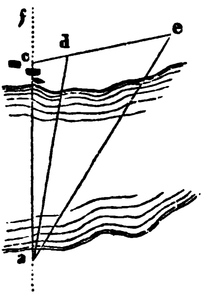
If the angle *d b c* exceeds 4°, and the distance *b c* is greater than 200 feet,—or even with an angle of 2½°, should the distance be greater than 300 feet,—*b c* will differ sensibly in length from *b k*, and a calculation of the latter becomes necessary. To effect this, multiply the natural cosine of the angle *k b c* by *b c*. This result *doubled* will give *b k d*, the length proper along tangent.

Thus:—Suppose *k b c*, the angle deflected, to have been 5°, and the distance *b c* 340 feet. Then ·9962, the natural cosine of 5°, multiplied by 340, gives 338·7 for the distance *b k*. Double this makes *b k d* = 677·4, and shows a difference of 2·6 feet between *b d* and *b c d*.

ARTICLE IV.

SHOULD THE OBSTRUCTION LIE ON THE OPPOSITE BANK OF A STREAM,

AND it is desirable on any account not to run the line from *d*, corresponding to *a d*, set the instrument at *a*, in tangent, and deflect clear of the obstacle to *d*. *Point* at *d*, deflect to *e*, and point also there—marking the angles *c a d*, *d a e*. Chain the base *d e*, and placing the transit at *e*, measure the angle *d e a*. Data are thus obtained sufficient for the calculation of the line *d a*. The object now is to find the point *c* and the angle *d c a*.



The angle *a d e* subtracted from 180° will supply the

angle $c\,d\,a$, so that in the smaller triangle we have obtained two angles and their included side. The distance $c\,d$, and angle $d\,c\,a$ readily follow. The transit standing at e, c is placed, of course, in the prolongation of the base $d\,e$, and the distance $c\,d$ is carefully set off with the rod. Moving the instrument to c, and turning the angle $e\,c\,f = 180° - d\,c\,a$, we are again in tangent.

Example.—Let $c\,a\,d = 6°$, $d\,a\,e = 35°$, $d\,e\,a = 42°$, and the base $d\,e = 200$ feet. Then in the triangle $d\,e\,a$ we have

Nat. sine $d\,a\,e = (\cdot 5736)$: nat. sine $d\,e\,a = (\cdot 6691)$ :: $d\,e = (200) : d\,a$,

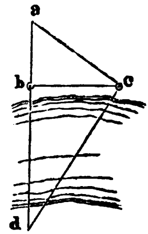
$$\text{Wherefore,} \quad d\,a = \frac{\cdot 6691 \times 200}{\cdot 5736} = 233 \cdot 3 \text{ feet.}$$

Again, the angle $c\,d\,a = 77°$. The angle $d\,a\,c$ being $= 6°$, $a\,c\,d$ is consequently $= 97°$, and in the small triangle we have, Nat. sin. $a\,c\,d = (\cdot 9925)$: nat. sin. $c\,a\,d = (\cdot 1045)$ :: $d\,a = (233 \cdot 3) : c\,d$.

$$\text{Therefore } c\,d = \frac{\cdot 1045 \times 233 \cdot 3}{\cdot 9925} = 24 \cdot 564 \text{ feet, and } d\,c\,f$$

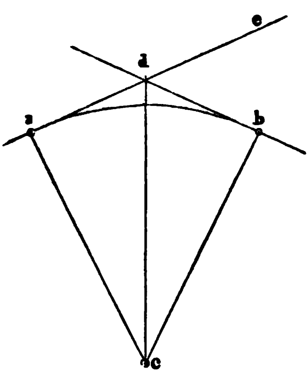
$180° - 97° = 83°$.

NOTE.—A common and convenient plan for triangulating a creek is as per figure. Set the instrument at b, fix a point d on the opposite shore, and making $d\,b\,c$ a right angle, place c at any convenient distance. Now move to c, sight to d, and making $d\,c\,a$ a right angle also, fix a, in the same line with b and d. a, c, and d are points in the circumference of a circle whose diameter is $a\,d$, $a\,b : b\,c :: b\,c : b\,d$, and therefore $b\,d = \frac{bc^2}{ab}$.



ARTICLE V.

HAVING GIVEN THE ANGLE *e d b*, FORMED BY THE INTERSECTION OF TWO STRAIGHT LINES, IT IS REQUIRED TO FIND THE POINT *a* OR *b*, AT WHICH TO COMMENCE A CURVE OF GIVEN RADIUS.



Draw the bisecting line *d c*. Then the angle *d c a* = half the angle *a c b* or its equal *e d b*; and in the triangle *d c a*, the angle *d a c* being a right angle, we have Rad. of 1 : Nat. Tang. *d c a* : Rad. *c a* : *a d*. Therefore *a d* = Nat. Tang. *d c a* × Rad. *e a*.

Example 1.—Let *e d b* = 48° and *a c* = 1460 feet. Here half the angle *e d b* or *a c b* = 24°, the Nat. Tang. of which is ·4452; and multiplying by Rad. 1460, we have 650 feet for the length of *a d* or *d b*, the tangents.

2.—*If a d be given and radius required,*—

$$\text{Rad.} = \frac{a\,d}{\text{Nat. Tang. } a\,c\,d} = \frac{650}{\cdot 4452} = 1460.$$

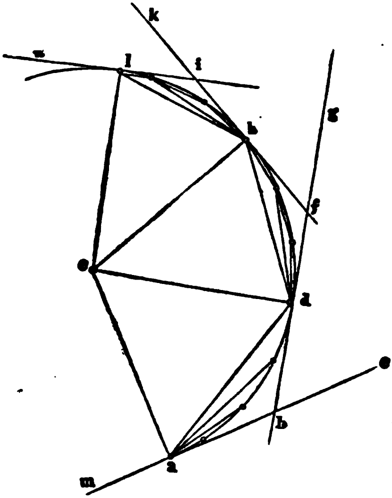
The following rules are approximate, and sufficiently correct for all purposes of location.

To find the degree of curvature of *a b* divide 5730 by the radius in feet; and to find the length of the curve in feet divide the angle *a c b* (after reducing minutes to hundredths) by the degree of curvature—the chord in each case being 100 feet in length.

ARTICLE VI.

TO TRACE A CURVE WITH TRANSIT AND CHAIN.

The degree of curvature and the angle to be turned are known. If the latter is expressed in degrees and minutes, reduce the minutes to hundredths, since the 100 feet chain is used, and divide the whole angle by the degree of curvature. The quotient will be the length of the curve in feet, and the P. T. is at once ascertained.



Let *m a* be the tangent, and *a* the P. C. Place the transit at *a*, index reading 0, and direct the sight along the tangent *m a e*. The first deflection will be half the central angle subtended by the chord used, and all the stakes put in from *a* will be fixed by similar tangential deflections. (*Prelim. Prop.* 1.)

When the point d is reached, the angle $d\,a\,b$, shown on the index, will be half the angle $d\,b\,e$, or its equal $a\,c\,d$, at the centre. Move the instrument to d, sight back to a, and turn to *double the index angle.* The telescope is now directed along the tangent $b\,d\,g$, and the angle $d\,b\,e = a\,c\,d = d\,a\,b + a\,d\,b$, reads on the index. Note this angle in the column of tangents opposite station d. Continue the curve from this new position, precisely as was done at a, and set the point h. Move to h, see that the vernier has not been disturbed, and sight back to d. The index now shows the angle $(d\,b\,e + h\,d\,g)$, and the object is to turn the angle $d\,h\,f$, i. e. *repeat* the angle $f\,d\,h$, as was done before at d, and, at the same time, have the whole angle $(d\,b\,e + h\,f\,g)$ indicated on the plate. To effect this, merely add this angle $f\,h\,d$ to the present reading. It will be found simpler, in practice, to double the entire angle thus far turned, and subtract from the product the last tangent, viz. $d\,b\,e$. The vernier, turned to this resultant angle, will put the telescope in tangent line to h. *And so on.*

Example.—"At sta. 24 + 50 commence a 4° curve to the left for 35° 12′." Suppose this a required duty. First, reducing minutes to hundredths, we have 35°·20, which, divided by 4°, gives 880 feet for the length of the curve. Adding 880 to 24 + 50 it is at once seen that sta. 33 + 30 is the P. T

Let a be the P. C., = 24 + 50. Now the *deflexion angle* being 4°, the *tangential* angle is 2°, with a chord of 100 feet. With a chord of 50 feet, therefore, the tangential angle is 1°, and this deflexion from tangent $m\,a\,e$ fixes station 25. A deflexion from this latter point of 2°, the chord being 100 feet, fixes station 26. *And so on.*

When you have fixed the point d, = sta. 28, the index reads 7°. Move up to station 28, sight back to the P. C., and turn the index to 14°. This throws you on tangent

Proceed as before, with the 2° deflexions, to sta. 31, $= h$. Move up, and sight back to sta. 28. The index now reads 20°. Multiplying by 2, and subtracting the last tangent, we have the reading of the tangent at $h = 26°$: we have turned 26° of the curve. Continue as before. After putting in sta. 33, to find the deflexion which shall fix the P. T., 33 + 30, say, as 100 feet : 30 feet : : 2° : the required deflexion, = 36′. We may here remark the great convenience of an instrument graduated to *hundredths* of a degree instead of *sixtieths*. In the present example it would be seen immediately that the tangential angle for 100 feet being 2°, for 1 foot it would be 2 *hundredths* of a degree, and for 30 feet it would be 60 hundredths.

Well! when the P. T., = 33 + 30, is fixed, the index reads 30° 36′. Move up, see that the vernier has not been disturbed, and sight back to sta. 31. Now twice the index reading, minus the last tangent, = 61° 12′ — 26°, = 35° 12′, the present tangent, which is the final tangent, which finishes the curve.

The advantage of this manner of running a curve is that the instrument shows at a glance the work done, and therefore errors may be detected with greater facility. By comparing at the P. T. the total index angle with the distance run, the work is tested at once.

The above is recorded in the field book as follows:—

Station.	Distance.	Deflex'n.	Index.	Tangent.	Course.	Mag. Course.	Remarks.
23∘	100ft.				N. 20°·00 W.	N. 20° 05 W.	
24	50						
P. C. + 50∘	50	0° 1·00	0° 1·00				At sta. 24 + 50
25	100	2·00	3·00				commence a 4°
26	100	2·00	5·00				curve to the L.
27	100	2·00	7·00				for 35° 12′.
28∘	100	2·00	16·00	0° 14·00		N. 34° 07′ W.	
29	100	2·00	18·00				
30	100	2·00	20·00				
31∘	100	2·00	28·00	28·00			
32	100	2·00	30·00				
33	30	0·36′	30·30′				
P. T. + 30∘	70			35·12′	N. 55°·12 W.	N. 55° 18′ W.	
34	100						

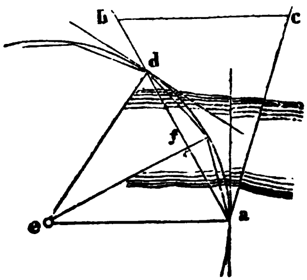
In running compound and reversed curves the operation is quite as simple as the foregoing. A point is fixed at the P. C. C., or P. R. C., and turning into tangent at that point, the second curve is traced from this *tangent*, without regard to what precedes. In reverse curving, it is a good plan to adjust the index in such manner at the P. R. C., that when we turn into tangent it will read 0.

This saves troublesome work, and it is advisable moreover to show in the field-book the contained angle *of each curve*, as well as the test of the two tangents with the magnetic course.

ARTICLE VII.

TO TRIANGULATE ON A CURVE.

SET the transit at *a*, and, as usual, sight back, and turn into tangent. Estimate the distance to the farther bank —do it liberally—and make a deflexion around the curve, corresponding to your estimated distance. Fix a point *b* in this line. Measure any convenient angle, *b a c*, and set



the point *c*. Move to *b*, measure the base *b c*, the angle *a b c*, and, before lifting the instrument calculate the line *b a*. If the angle turned from tangent to *d* exceeds 4°, and the distance is greater than 200 feet, the chord *a d* must also be calculated, as per example, and the difference between this and *b a* will be the distance *b d* to the point *d*, in the curve, which can be fixed from *b*.

Should b fall between d and a the operation is analogous.

Example.—Let a be a point in a 6° curve. Having set the transit, and turned into tangent, the distance to the farther verge is estimated 400 feet. The tangential angle for 100 feet is 3°, and to fix d, 400 feet distant, is consequently 12°. Deflect this angle, fix a point in line, and complete the triangulation, as previously illustrated in Art. IV., p. 22. Suppose $a\,b$ found equal to 472 feet. Now the tangential angle to d = half the central angle, $= f\,e\,a$, = 12°; and to find the length of the chord $a\,d$, we have, in the triangle $e\,f\,a$,

Rad. : Sin. $a\,e\,f$: : $e\,a$: $a\,f$, that is

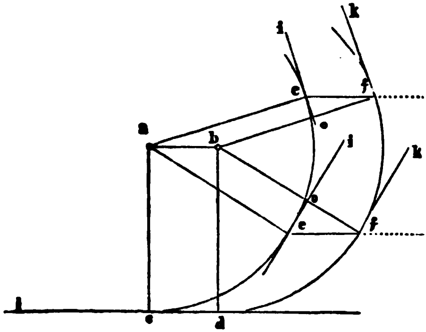
Rad. of 1 : Nat. Sin. 12° : : 955·4, the Rad. of the 6° curve : half the chord required. Wherefore $a\,d$ = twice the Nat. Sin. 12° × 955·4 = ·2079 × 2 × 955·4 = 397·25 feet. Subtracting this from 472, we have the distance, 74·75 feet, back to the point in the curve. Move the instrument to d, set the index at 12°, sight back to a, and turning to 24°, the telescope is in tangent. A deflexion of 3° will fix the next station.

NOTE.—In this case, if preferred, a third proportional might be formed with the chord of crossing, as shown in the note to Art. IV.

ARTICLE VIII.

TO CHANGE THE ORIGIN OF ANY CURVE, SO THAT IT SHALL TERMINATE IN A TANGENT PARALLEL TO A GIVEN TANGENT.

Let *d f* be the located curve, terminating in a tangent *f k*, and the nature of the ground requires that it should terminate in the tangent *e i*, parallel to *f k*. At *f*, the telescope being directed along the tangent *f k*, turn to the

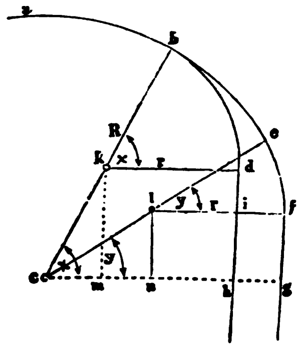


right an angle equal to the central angle *d b f*, previously turned to the left on the curve. This will direct the telescope along *e f*, parallel to *d l*. Measure *e f*, and go back on the tangent, *d l*, a distance, *c d*, equal to it. The curve, retraced from *c* and consuming the same angle, will termi nate tangentially in *e i*. An example in this case is not necessary.

ARTICLE IX.

TO CHANGE A P. C. C. SO THAT THE SECOND CURVE SHALL TERMINATE IN A TANGENT PARALLEL TO A GIVEN TANGENT.

Let $a\,b\,d$ be the compound curve, located and terminating in the tangent $d\,h$. Continue the larger curve to e, and from e, with radius $e\,l = k\,b$, describe the curve $e\,f$, terminating tangentially in $f\,g$, parallel to $d\,h$. From c, the centre of the larger curve, let fall upon $f\,g$ the perpen-



dicular $c\,g$, and fill up the figure as above. Call the radii respectively R and r, the angle $b\,k\,d$, or its equal, $k\,c\,m$, x, and the angle $e\,l\,f$, or its equal, $l\,c\,n$, y. Let the distance $i\,f$, or $h\,g$, be named D. Now the line $c\,g$ is made up of the lines $c\,m + m\,h + h\,g$, i. e., $c\,g = \text{cosin. } x\,(\text{R} - r) + r + \text{D}$. $c\,g$ is also made up of the lines $c\,n + n\,g$, i. e., $c\,g = \text{cosin. } y\,(\text{R} - r) + r$. Therefore $\text{cosin. } x\,(\text{R} - r) + r + \text{D} = \text{cosin. } y\,(\text{R} - r) + r$, and reducing,

$\text{cosin. } y = \dfrac{\text{cosin. } x\,(R - r) + D}{(R - r)}$; so that the distance *i f*, or *h g*, measured rectangularly between the two tangents, being added to the nat. cosin. x, will give the nat. cosin. of the angle *e l f*, to be turned on the smaller curve. The angle y, subtracted from the angle x, gives of course the angle *b c e*, to be advanced on the larger curve; or, dividing this angle by the degree of curvature of *a b*, we find the distance from *b* to *e* the P. C. C. proper.

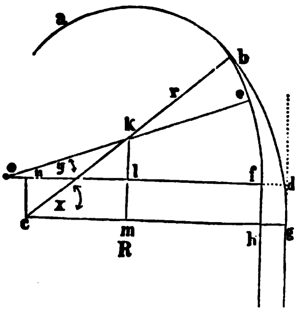
If *e f* be the second curve located, and the tangent to be touched lies *within*, it is evident that we must *retreat* upon the large curve, and, by *subtracting* D from the cosine of the angle y, we obtain the cosine of the angle x.

Example.—Suppose *a b* a 3° curve located, and compounding, at *b*, into a 6° curve, which latter is continued to the right through an angle of 42°. At the P. T. we discover that the proper tangent is 64 feet to the left. We must throw our curve *out*, then—we must *advance* on the 3° curve a certain distance. How to find this distance: The radius of a 3° curve = 1910; the radius of a 6° curve = 955·4; R — r, therefore, = 954·6. The nat. cosin. 42° = ·7431. Now, by the formula just obtained, we have $\dfrac{\text{cos. } x\,(R - r) + D}{(R - r)} = \dfrac{(\cdot 7431 \times 954\cdot 6) + 64}{954\cdot 6} =$ ·8101 = nat. cosin. 35° 53′. Subtracting this from 42°, we have 6° 07′, the angle to be advanced on the 3° curve; or, reducing minutes to hundredths, and dividing by 3°, we find 204 feet, the distance from *b* to the correct P. C. C

ARTICLE X.

SHOULD THE SECOND CURVE BE ONE OF LONGER RADIUS THAN THE FIRST,

OUR illustration takes simpler form, and the application of D varies *vicê versa.*



See figure, analogous to that of the previous problem. Here $o\,f$, $c\,g$ are equal and parallel radii; $f\,h$ a perpendicular connecting them. Draw its fellow, $c\,n$. Then, $n\,f = c\,h$, and, consequently, $h\,g = o\,n$. Again, letting $k\,m$ fall, perpendicular to $c\,g$, we have $c\,m = n\,l$, and $o\,l = c\,m + o\,n$; i. e., cosin. y (R — r) = cosin. x (R — r) + D.

We observe that, with a curve of this nature, in order to throw the line *farther out*, it is necessary to *go back*, toward b; or, having located to g, if the object tangent lie *within*, we must *advance* toward e.

Example.—Suppose $a\,b$ a 5° curve, b the P. C. C., and $b\,g$ a 2° curve. Setting the instrument at g, the P. T., and turning into tangent, we find that we are a distance $h\,g$, = 53 feet, too far to the *left*. The first question is, what angle have we turned on the second curve. Let it

be 28°. Now we know, that, in order to strike farther to the *right*, we must advance on the 5° curve. Consequently, D must be *added* to the cosine of 28°, to give us the cosine of the proper angle for the 2° curve; and the difference between 28° and this newly found angle will be the angle we are to advance on the 5° curve. Thus:—the rad. of a 5° curve = 1146 feet, that of a 2° curve = 2865 feet and their difference = 1719 feet. The nat. cos. of 28° = ·8829. Then $\frac{(\cdot 8829 \times 1719) + 53}{1719} = \cdot 9137$, = nat. cos. 23° 58′. This, subtracted from 28°, leaves 4° 02′, = 80 feet, from *b* to the correct P. C. C.

Synopsis of the preceding formulæ.

Call D the distance between tangents as before, *a* the angle of the second curve located, and *b* the angle of the same curve to be substituted for it.

FIRST, when the second curve has the *smaller* radius—

Tangent falling *within* the point, cosine $b = \frac{\cos. a\,(R - r) + D}{(R - r)}$.

Tangent falling *without* the point, cosine $b = \frac{\cos. a\,(R - r) - D}{(R - r)}$.

SECOND, when the second curve has the *larger* radius—

Tangent falling *within* the point, cosine $b = \frac{\cos. a\,(R - r) - D}{(R - r)}$.

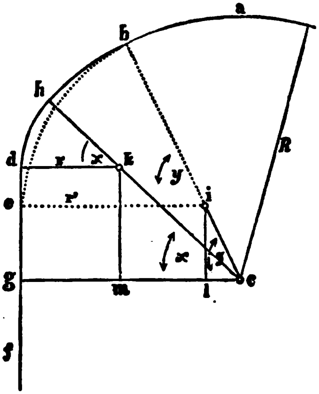
Tangent falling *without* the point, cosine $b = \frac{\cos. a\,(R - r) + D}{(R - r)}$.

Very little attention will familiarize these formulæ, and render the field practice easy.

ARTICLE XI.

HAVING LOCATED THE COMPOUND CURVE *a b d*, TERMINATING IN THE TANGENT *d f*, IT IS REQUIRED TO FIND THE P. C. C. *b*, AT WHICH TO COMMENCE ANOTHER CURVE OF GIVEN RADIUS, WHICH SHALL ALSO TERMINATE TANGENTIALLY IN *d f*.

PLOT the curves as per figure. From *c* let fall *c g*, perpendicular to the tangent, *d f*. From *k* and *i*, the lesser centres, drop *k m*, *i l*, perpendicular to *c g*. Call the great radius R, the smaller radius *r*, and the *intended* radius of



the second curve r'. Likewise name *h k d*, the angle of the small curve located, x, and *b i e* the angle to be found for the *proposed* curve, y. Now, the tangent *d f*, and the curve *a b*, lying unstirred, the line *c g* is an unvarying distance, and it is made up of the lines $c m + m g$, i. e., $c g = (R - r) \text{ cosin. } x + r$. It also consists of the lines *c l*

$+ l g$, i. e., $c g = (R - r')$ cosin. $y + r'$, and, reducing, nat. cosin. $y = \frac{(R - r) \text{ cosin. } x + r - r'}{(R - r')}$. This, therefore, is the formula by means of which we can ascertain the point b, as follows:—

Example.—Imagine a 2° curve, $a h$, compounding into a 6° curve, $h d$, which terminates at d, in the tangent $d f$. The tangent lies well; the curve $a h$ likewise; but it is desired to throw the line to the left, on better ground, between d and h, by means of an intercalary 4° curve. We wish, then, to know the distance, $h b$, back to the new P. C. C.

The radius of a 2° curve = 2865 feet, of a 6° curve = 955·4 feet, and their difference $(R - r) = 1909{\cdot}6$. The radius of a 4° curve = 1433, and the difference $(R - r')$ is, therefore, 1432 feet. Let $h k d$, the angle turned on the 6° curve, be 41°, the nat. cos. of which = ·7547.

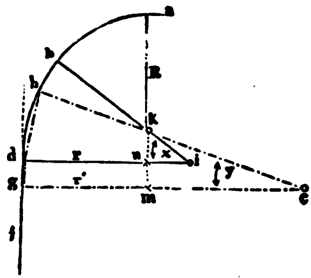
Then, $\frac{(\cdot 7547 \times 1909{\cdot}6) + 955{\cdot}4 - 1433}{1432} = \cdot 6728,$ = nat. cosin. 47° 43′. Subtracting 41°, we have 6° 43′, the angle $h c b$. Reducing minutes to hundredths, and dividing by 2°, we find 336 feet to be the distance from h to b. A 4° curve of 47° 43′, traced from this latter point, will terminate in the tangent $e f$.

ARTICLE XII.

IF THE LATTER CURVES HAVE LARGER RADII THAN THE FIRST,

THE solution retains its shape and simplicity.

Draw the figure as above, and, for the sake of uniformity, name the radii as before. The curve $a\,b$, and tangent $d\,f$, being constant, the distance $m\,g$, or $d\,n$, is here constant. Call it A. Now A, in the first place, is equal to $d\,i - n\,i$, i. e., $= r - (r - \text{R})$ cosin. x; and, in the second place,



it is equal to $g\,c - m\,c$, $= r' - (r' - \text{R})$ cosin. y; wherefore $r - (r - \text{R})$ cosin. $x = r' - (r' - \text{R})$ cosin. y, and consequently cosin. $y = \dfrac{(r - \text{R}) \text{ cosin. } x + r' - r}{(r' - \text{R})}$.

Example.—Suppose $a\,b$ a 7° curve, compounding, at b, into a 5° curve, $b\,d$, which latter subtends an angle of 38°, and terminates in the tangent $d\,f$. We wish to substitute a terminal 2° curve, $h\,g$, and to know the position, h, of the new P. C. C.

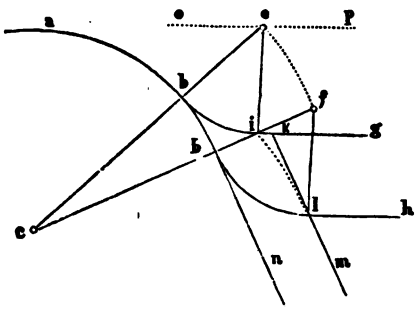
The radius of a 7° curve $= 819$ feet $=$ R. r and r',

the radii respectively of 5° and 2° curves, are equal to 1146, and 2865 feet. $r - R$, therefore, $= 327$, and $r' - R = 2046$ feet. The nat. cosin. of 38° $= \cdot 788$. Then, by the formula, $\frac{(\cdot 788 \times 327) + 2865 - 1146}{2046} =$ $\cdot 9661$, $=$ the nat. cosin. of 14° 57′, the angle to be turned on the 2° curve. Subtracting this from 38°, we have 23° 03′, the angle to be continued on the 7° curve. Reducing minutes to hundredths, and dividing by the degree of curvature, 7°, we find 329 feet, the distance from *b* to the new P. C. C. *h*.

ARTICLE XIII.

TO CHANGE A P. R. C., SO THAT THE SECOND CURVE SHALL TERMINATE IN A TANGENT PARALLEL TO A GIVEN TANGENT.

Let *a d l* be the reverse curve, located and terminating



in the tangent *l h*. Call the radius *c b*, R, and the radius *b e*, *r*. Suppose *i g* the given tangent. At a distance

from it equal to $i\,e$, the radius of the second curve, draw the parallel line, $o\,p$. With c as a centre, and radius $c\,f$, $= \mathrm{R} + r$, describe the integral curve, $f\,e$, cutting $o\,p$ in e. e, then, is the centre of the curve adjusted.

Application.

Place the transit at the P. T., l, and turn into a tangent, $l\,m$, parallel to $d\,n$, the common tangent of the two curves at d. Unless some wide mistake has been made, the distance $l\,k$, measured along this line to $i\,g$, the tangent proper, will be about equal to the distance $e\,f$, and we shall have the proportion, $c\,f : f\,e :: c\,d : d\,b$, i. e., $\mathrm{R} + r : e\,f :: \mathrm{R} : d\,b$, which gives $\frac{f\,e \times \mathrm{R}}{\mathrm{R} + r}$, as a simple formula for finding the distance back from d to b, the correct P. R. C. This rule, though sufficiently true for most cases, is not mathematically justifiable. It will be seen that $e\,f$, or its equal $i\,l$, the distance we wish to measure, is a *curving* distance, part of the circumference of a circle concentric with $a\,b$. Its radius is $(\mathrm{R} + r)$, therefore its degree of curvature $= \frac{5730^\circ}{(\mathrm{R} + r)}$, or, more simply, equals the *product* of the degrees of curvature of the curves composing the reverse, divided by their *sum*. To be strictly accurate, then, set the instrument at l, turn into tangent $l\,m$ as before, and trace the curve $i\,l$, until it strikes the tangent $i\,g$. The angle which $i\,l$ subtends, being divided by the degree of curvature of $a\,b$, will give the distance, $d\,b$, to the P. R. C. proper. The curve retraced from b, will terminate tangentially in $i\,g$, and its angle, $b\,e\,i$, will be equal to $d\,f\,l$ — $d\,c\,b$.

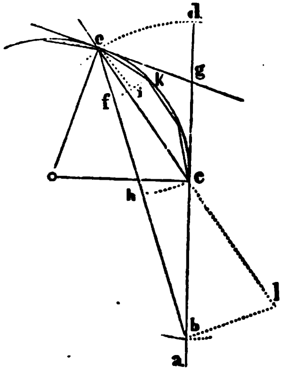
Example.—Let $a\,d\,l$ be a reverse curve, composed of a 3° curve, $a\,d$, and a 6° curve, $d\,l$. Let the angle $d\,f\,l$ be equal to 52°, and suppose the distance $l\,f$ to have been

found 34 feet. Being part of a 2° curve, it therefore subtends a central angle of 41′. This corresponds to a distance of 23 feet, to be gone back on the 3° curve, and 52°·00 — 41′ = 51° 19′, the angle to be turned from *h* on the 6° curve, in order to strike the tangent *i g*.

ARTICLE XIV.

HOW TO PROCEED WHEN THE P. C. IS INACCESSIBLE.

In the figure, drawn to illustrate this case, let *c* be the point of curvature, *c a* the tangent, and *c k e* the curve. Now the angle *d c e*, included between the tangent and



any chord, as *c e*, fixing the point *e*, is known. Make *c b* along tangent, equal to *c e*, and connect *b e*. If a circle were now described from *c* as a centre, with radius *c e* or *c b*, *d*, *e*, and *b*, would be points in its circumference, and

the angle $d\,b\,e$ at once proven equal to half the angle $d\,c\,e$. With proof precisely similar, $d\,c\,e =$ half of $d\,g\,e$, and, consequently, $d\,b\,e$ is equal to one-fourth of the central angle subtended by the chord $c\,e$.

Example.—Suppose c to be the inaccessible point of curvature of a 6° curve, $c\,k\,e$. It is concluded to run to the third station, e. First we must calculate the length of the chord $c\,e$. The angle $d\,c\,e = 9°$, and from Art. VII. we have

Rad. of 1 : 955·4 :: nat. sin. 9° $=$ ·1564 : $\frac{e\,c}{2}$, whereby $e\,c$ is shown equal to 298·8. Place the transit then at b, 298·8 feet distant from the P. C., and deflect to the left an angle of 4° 30′, equal to half the angle $d\,c\,e$. This is in line to e, and $b\,e$ must likewise be calculated as follows:

In the triangle $b\,c\,h$ we have

Rad. of 1 : nat. cosin. 4° 30′ $=$ ·9969 :: $b\,c =$ 298·8 : $b\,h = \frac{b\,e}{2}$, whereby $b\,e$ is shown equal to 595·7 feet. Arriving at e, the index reads 4° 30′. Sight back to b, turn to 18°, and the telescope will be in tangent. Suppose, however, that having reached f, 100 feet from e, this latter point is also found inaccessible. We find k a different point in the curve, thus:—The angle $f\,e\,g = 18° - 4°\,30' = 13°\,30'$, and the tangential angle $g\,e\,k = 3°$. Consequently the angle $f\,e\,k = 10°\,30'$, and, drawing the bisecting line $e\,i$, we have, in the triangle $e\,f\,i$,

Rad. of 1 : nat. sin. 5° 15′ $=$ ·0915 :: $e\,f = 100$: $f\,i$ $=$ 9·159 feet. Therefore $f\,k =$ 18·318 feet, and the angle $e\,f\,k = 90° - 5°\,15' = 84°\,45'$. At f deflect this angle to the right, and measure the distance $f\,k$ carefully with the rod. At k, sighting back to f, and turning the equal angle $f\,k\,e$, the telescope will be directed to e, and the curve may be continued.

If it is inconvenient to run the line $b\,e$, the point e may

be reached thus:—Fix the P. C. Find the tangential distance *d e*, corresponding to the angle *d c e*. Carefully with the rod lay off *b l*, equal to it, at right angles to *b e* Set the transit at *l*, and, in line with *c*, put in *e*.

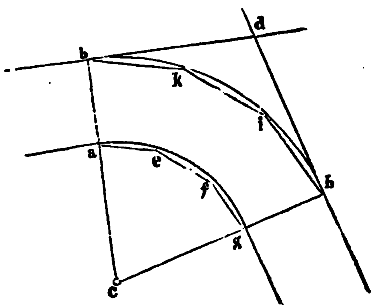
The distance *b l* should not exceed 10 or 12 feet.

NOTE.—The foregoing illustrations will apply when the P. T. is likewise inaccessible.

ARTICLE XV.

TO AVOID OBSTACLES IN THE LINE OF CURVE.

LET *b k h* be the curve. We can either follow the tangents *h d*, *d b*, or trace a parallel curve, *g a*, *within* the first; which tracing is effected thus:—Set the instrument



at *h*, the P. C., and offset any distance *h g*, at right angles to the tangent *h d*. It will be observed that as the distance *h g increases*, the distance *g a decreases*, whilst the angle subtended by *g a* remains equal to that subtended

by $h\ b$; i. e., our *deflexions* on the offset curve stand unchanged, but the corresponding *chords*, $g\ f$, $f\ e$, &c., are *less* than their equivalents, $h\ i$, $i\ k$, &c., along $h\ b$. To find their length, $h\ i$, $i\ k$, &c., being equal to 100 feet, we have the proportion, $c\ h : c\ g :: h\ i : x$; i. e., R : rad. — $h\ g$ :: 100 feet : x, where x symbols the unknown chord. Now set the transit at g, turn into tangent parallel to $h\ d$, and with the shortened chord, fix $f\ e\ a$. Rectangularly to the tangents at these points, and distant $h\ g$, will be i, k, b of the curve proper.

Example.—Let $b\ h$ be a 4° curve, and the offset distance 85 feet. The radius then is 1433, and

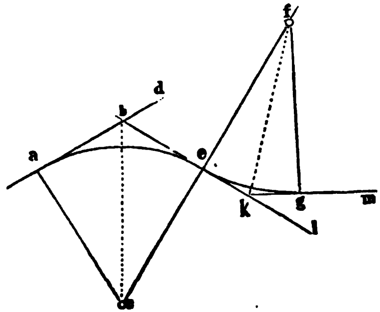
1433 : 1348 : : 100 : 94, the short chord.

To follow the tangents, suppose the angle $b\ c\ h$ = 42°. Then by Art. V. we find the tangent $h\ d$ = 550 feet. which distance we duly measure, and, at d, deflecting 42°, lay off an equal distance to b, the point of tangency.

ARTICLE XVI.

HAVING GIVEN THE ANGLES $d\,b\,k$, $m\,k\,l$, AND THE DISTANCE $b\,k$, IT IS REQUIRED TO FIND THE RADII $c\,e$, $e\,f$ OF THE EASIEST REVERSE CURVE WHICH SHALL UNITE $a\,d$, $k\,m$.

THE angle $d\,b\,e$ is equal to the angle $a\,c\,e$, half of which is $b\,c\,e$. So likewise $e\,f\,k$ is equal to half of $l\,k\,m$.



Then, [nat. tang. $b\,c\,e$ + nat. tang. $e\,f\,k$] : nat. tang. $b\,c\,e$ :: $b\,k$: $b\,e$, and $b\,k - b\,e = e\,k$. Wherefore rad. $c\,e$

$$= \frac{b\,e}{\text{nat. tang. } b\,c\,e} \text{ and rad. } e\,f = \frac{e\,k}{\text{nat. tang. } k\,f\,e}.$$

Example.—Suppose the angle $d\,b\,e = 54° \; 30'$, the angle $l\,k\,g = 33° \; 20'$, and the distance $b\,k = 832$ feet.

Therefore the angle $b\,c\,e = 27° \; 15'$, the nat. tang. of which is ·5150, and the angle $e\,f\,k = 16° \; 40'$, the nat. tang. of which is ·2994. The sum of the tangents = ·8144. Then, to find $b\,e$, we have

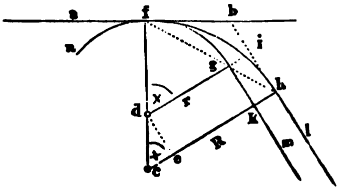
As ·8144 : ·5150 :: 832 : 526, and subtracting this from $b\,k$, we have $e\,k = 306$ feet.

Again, the radius $c\,e = \dfrac{526}{\cdot 515}$, and the radius $e\,f = \dfrac{306}{\cdot 2994}$, = 1022 feet.

ARTICLE XVII.

HAVING GIVEN THE CURVE fg, LOCATED AND TERMINATING IN THE TANGENT $g\,m$, IT IS REQUIRED TO FIND WHERE A CURVE OF DIFFERENT RADIUS WILL TERMINATE IN A TANGENT PARALLEL TO $g\,m$.

LET fg be the curve located, and fh the curve proposed, commencing at the common point f, and terminating in the parallel tangents, $g\,m$, $h\,l$. We wish to find the length and direction of the line $g\,h$, connecting the points of tangency.



Call the radii respectively R and r, and the central angle, $f\,d\,g$ or $f\,c\,h$, x. Now $g\,k = d\,e$, = sine of x, $c\,d$ being radius; i. e., = nat. sin. $x \times (\mathrm{R} - r)$. Again, the angle $k\,g\,h = f\,h\,b = \frac{x}{2}$, and $g\,k$ = cosin. $k\,g\,h$, $g\,h$ being radius, i. e., $g\,k$ = nat. cos. $\frac{x}{2} \times g\,h$, and, consequently. nat. cos. $\frac{x}{2} \times g\,h$ = nat. sin. $x \times (\mathrm{R} - r)$, wherefore

$$g\,h = \frac{(\mathrm{R} - r)\ \text{nat. sin. } x}{\text{nat. cos. } \frac{x}{2}}.$$

It will be observed that the angle $k\,g\,h$, included between

the tangent to a curve at any point g, and the line $g\,h$ connecting g with an equivalent point h in any other curve $f\,h$ commencing at the same P. C., f, and turning in the same direction, is invariably equal to half the common central angle, $f\,d\,g$ or $f\,c\,h$.

Example.—Let $f\,g$ be a 7° curve, subtending a central angle, $f\,d\,g$, of 44° 26'. Having arrived at g, the P. T., and turned into tangent, $g\,m$, it is desired to fix the P. T., h, of a 4° curve which shall likewise subtend an angle of 44° 26'. Here the radii are, respectively, 819 and 1433 feet, and $R - r = 614$. The nat. sine of $44° 26' = \cdot700$, and the nat. cosine of half this angle, viz.: $22° 13' = \cdot9258$. Then, by the formula, $g\,h = \dfrac{614 \times \cdot7}{\cdot9258} = 464\cdot2$.

Deflecting, therefore, to the left, an angle of 22° 13', and laying off the distance 464·2 feet, we arrive at the point h. Move to h, sight back to g, and a deflexion of 22° 13' to the right will direct the telescope along the tangent $h\,l$.

If h were the P. T. located, and g the point required, the same anglo and distance would apply.

ARTICLE XVIII.

HAVING THE CURVE *n f g* LOCATED, AND TERMINATING IN THE TANGENT *g m*, IT IS REQUIRED TO FIND THE POINT *f*, WHEREAT TO COMPOUND WITH ANOTHER CURVE OF GIVEN RADIUS, WHICH SHALL TERMINATE IN *i l*, PARALLEL TO *g m*.

[See previous figure.]

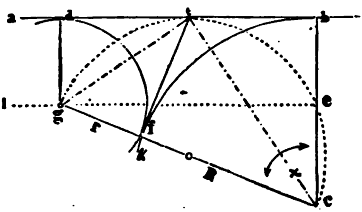
NAME the radii and angle as before. Measure the distance *g i*, between the tangents, and call it D. Then *c h* is equal to $c\,e + e\,k + g\,i$ or $k\,h$; i. e., $R = (R - r)$ cosin. $x + r + D$, or, transposing, cosin. $x = \frac{R - (r - D)}{(R - r)}$. Thus discovering the angle *f d g*, divide it by the curvature *n f g*, and we have the distance, *g f*, to the P. C. C. *f*. The second curve, traced from this point, will terminate tangentially in *h l*.

Example.—Let the curvatures equal those of the last problem, and suppose the distance *g i* to be 175·6 feet. Then, by the formula, cosin. $x = \frac{1433 - 994{\cdot}6}{614} = {\cdot}7143$ $=$ cosin. 44° 26′. Reducing minutes to hundredths, and dividing by 7°, we have 635 feet, the distance back to the P. C. C.

ARTICLE XIX.

HAVING GIVEN A TANGENT $a\,b$, AND A CURVE $b\,k$, LOCATED, IT IS REQUIRED TO FIND THE POINT d, OR f, AT WHICH TO COMMENCE A CURVE OF GIVEN RADIUS, WHICH SHALL BE TANGENT TO BOTH.

DRAW the radius $b\,c$, and call it R. Name the radius of the other curve r. Make $b\,e$ equal to r, and, through e, draw the line $e\,l$, parallel to the tangent $a\,b$. From c as a centre, with $c\,g$, $= (R + r)$, as radius, sweep the arc of a circle; which arc will intersect $e\,l$ at g, and, from the equidistances, prove g the centre of the other curve touching $a\,b$, $b\,k$, tangentially, at the points d and f.



Now, to find the point f for purposes of location, we must know the angle $b\,c\,f$. Call it x. In the triangle $c\,g\,e$, we have $c\,g : c\,e ::$ radius : cosin. $g\,c\,e$, or $b\,c\,f$; i. e., $(R + r) : (R - r) :: 1$: nat. cosin x, wherefore nat. cosin. $x = \frac{(R - r)}{(R + r)}$. So that, having divided the *difference* of the radii by their *sum*, we shall find opposite to the quotient, in the table of nat. cosines, the angle $b\,c\,f$ required. This angle, divided by the degree of curvature of $b\,k$, will

give the distance from b to the P. R. C., f; and $180° - x$ will equal the angle $f g d$, to be turned on the other curve.

Another plan, which may be preferred, for finding the point d, is as follows:

Make $c g$ the diameter of a semicircle, $c t g$. This semicircle is tangent to $b d$ at t, and $t f$, perpendicular to $c g$, is a common tangent to the curves $b k$, $d f$. We have then, $cf \times fg = tf^2$, i. e., $R \times r = \text{tang.}^2 \frac{x}{2}$. Multiplying the radii together, and extracting the square root of the product, we find the distance $t f$, or its equal $t b$, which, *doubled*, is the distance $b d$, from b to the point of curvature, d.

Example.—Suppose $k b$ a 3° curve, tangent to the line $a b$. We wish to know the point f, at which to begin a 5° curve, which shall be tangent to both. Here $R = 1910$, and $r = 1146$, wherefore $(R + r) = 3056$, $(R - r) = 764$, and $\frac{(R - r)}{(R + r)} = \frac{.764}{3056}, = \cdot 25, =$ nat. cosin. 75° 31′. Reducing minutes to hundredths, and dividing by 3°, we have 2517 feet, the distance from b to the P. R. C., f.

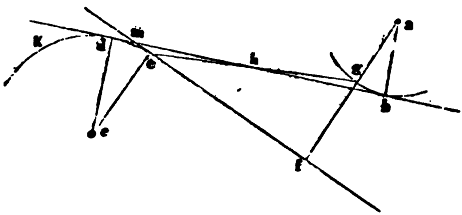
If the point d be required, we have $\sqrt{1910 \times 1146} = 1478 \cdot 9$. Doubling this result we find $2957 \cdot 8$ feet, the distance from b to the P. C., d.

If the radii are equal, of course the distance $b d$ is equal to their *sum*, and the angle x is a right angle.

ARTICLE XX.

TO RUN A TANGENT TO TWO CURVES.

LET $g\,b$, $e\,k$, be the two curves. First plot them carefully to a large scale, and, finding from this plot an approximate P. T., say e, run the tangent ef. Now the chord in the curve $g\,b$, to which ef is parallel, is known, and consequently the shortest distance, fg, between the tangent and the curve may be measured. Then $ef : fg$ :: rad. : tang. $f\,e\,g$, and nat. tang. $f\,e\,g = \frac{fg}{ef}$. *Practically*, this angle $f\,e\,g$ will be found equal to $g\,a\,b$ or $d\,c\,e$, so that, dividing it by the degree of curvature of $k\,e$, we shall have the distance, $e\,d$, to the proper P. T.



Strictly speaking, the angle $g\,e\,f$ is too great by the small angle $b\,h\,g$. There are two modes of calculating this latter, but they are both complex, and in any but a most unusual case, the above rule is sufficiently accurate.

Example.—Let $k\,e$ be a 5° curve. Suppose the tangent $ef = 1632$ feet, and the distance $gf = 42$ feet,

Then $\frac{42}{1632} = \cdot 0257 =$ nat. tang. of 1° 28′. Reducing

minutes to hundredths, and dividing by 5°, we have 29 feet, the distance back to the correct P. T.

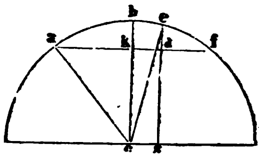
Should the curves turn in the same direction, or should *e* be a fixed point from which to run a tangent to *g b*, the above illustration is still applicable.

ARTICLE XXI.

ORDINATES.

TO FIND THE MIDDLE ORDINATE TO ANY GIVEN CHORD, IN A CURVE OF ANY GIVEN RADIUS.

1st. (See figure in Art. XVI.) $k\,c = \sqrt{a\,c^2 - a\,k^2}$, and $a\,c$ or $b\,c - k\,c =$ the ordinate required. That is, from the square of the radius subtract the square of half the chord, and take the square root of the remainder from radius, for the middle ordinate.



Example.—The radius $a\,c$ being 819 feet, and the chord $a\,f$ 100 feet, to find the middle ordinate, $b\,k$.

Here $a\,c^2 - a\,k^2 = 670761 - 2500, = 668261$, the square root of which is $c\,k, = 817{\cdot}5$, which taken from radius 819, leaves $1{\cdot}5$, the required middle ordinate.

2d. Subtract the nat. cosine of the tangential angle from 1, and multiply the remainder by radius.

Example.—Suppose $a\,b$ a 7° curve. Here the nat. cosine of 3° 30′, the tangential angle, is ·9981, which, subtracted from 1, leaves ·0019. Multiplying this latter by radius 819, we have 1·5, the middle ordinate as before

HAVING GIVEN THE MIDDLE ORDINATE, TO FIND ANY OTHER.

1st. $e\,g = \sqrt{c\,e^2 - c\,g^2}$, and $e\,d = e\,g - g\,d$ or $c\,k$.

Example.—Suppose the distance $k\,d$ or $c\,g$ to be 20 feet. Thence $c\,e^2 - c\,g^2 = 670761 - 400 = 670361$, the square root of which is $e\,g$, $= 818·76$. Taking from this the distance $g\,d = 817·5$, we have the ordinate $e\,d = 1·26$.

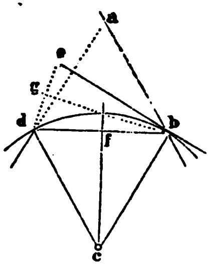
2d. Multiply the ordinates of a 1° curve by the deflexion angle of the curve whose ordinates are required. This is an approximation sufficiently exact for railway curves.

ARTICLE XXII.

TO FIND THE RADIUS CORRESPONDING TO ANY DEFLEXION ANGLE, AND TO EQUAL CHORDS OF ANY GIVEN LENGTH.

HERE the deflexion angle, $d\,b\,a$, is of course equal to the central angle $d\,c\,b$, subtended by the given chord $d\,b$, and the triangle $c\,d\,b$ being isosceles we have $\frac{180° - d\,c\,b}{2}$ $= c\,d\,b$, or $d\,b\,c$.

Then nat. sin. $d\,c\,b$: nat. sin. $c\,b\,d$ :: $d\,b$: $c\,d$, the radius.



Example.—Let the deflexion angle be 5°, and the chord 100 feet. Required the radius. The nat. sine of 5° = ·0872 and $\frac{180° - 5°}{2} = 87° \, 30'$, the nat. sine of which is ·9990.

Then ·0872 : ·9990 :: 100 : radius = 1146 feet.

An approximate result, sufficiently accurate for all purposes of railway location, may be found by simply dividing 5730 by the deflexion angle.

To find the deflexion angle corresponding to any given

radius and chord.—In this case, we have $c\,d : d\,f :: $ rad of 1 : nat. sin. of half the deflexion angle; therefore, divide half of the chord by the radius of the curve; the quotient will be the nat. sine of half the deflexion angle.

Example.—Radius as before 1146 feet, and chord 100 feet. Then $\frac{50}{1146} = \cdot 0436$, the nat. sin. of 2° 30′. Doubling this result we have the deflexion angle, viz. 5° 00′.

To find the deflexion distance with chord of 100 *feet and any radius.*—Divide the constant number 10000 by the radius in feet; the quotient will be the deflexion distance:—for the deflexion distance with a radius of 10000 feet is 1 foot, and the deflexion distances for other radii increase inversely as the radii.

Example.—What is the deflexion distance for a 5° curve, the chord being 100 feet? Here $\frac{10000}{1146} = 8 \cdot 72$ feet, the deflexion distance.

To find the deflexion distance with any given chord and radius.—The deflexion distance is equal to twice the natural sine of half the *deflexion* angle, multiplied by the chord. Thus, the chord being 100 feet, and the deflexion angle 5°, we find the nat. sine of 2° 30′ equal to ·0436, which doubled, and multiplied by 100, gives 8·72 feet, as above.

The tangential distance, with any radius and chord, is in like manner equal to twice the nat. sine of half the *tangential* angle multiplied by the chord. Thus, the tangential angle being 2° 30′ and the chord 100 feet, we find the nat. sine of 1° 15′ = ·0218. Multiplying by 2, and 100, we have 4·36 feet, the tangential distance.

For all curves under 11°, the tangential distance is equal to half the deflexion distance.

ARTICLE XXIII.

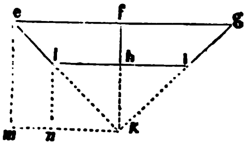
OF EXCAVATION AND EMBANKMENT.

A RAILWAY line having been located, the first office duties are to map it down, to make a continuous profile, and an approximate estimate of the cost of grading, &c. The "elevation" of each stake above or below a certain assumed base being fixed, the profile is drawn, on paper prepared for the purpose, to a horizontal scale of 400 feet, and a vertical scale of 40 feet, to the inch. This distorted picture presents at a glance, in compact shape, the undulations of the surface, and from it grades are adopted, to balance as nearly as possible the excavation and embankment. By these grades, noted in the record, the cutting or filling is ascertained at each station. Suppose, for instance, that the elevation of station 40 is 12 feet, and that of station 100 is 72 feet. The distance between 40 and 100 is 6000 feet, and the difference between 12 and 72 is 60 feet. Consequently, to connect those points, we require an ascending grade of 1 in 100 feet. Grade at station 54 is therefore 26 feet, and at station 60, 32 feet. If the elevation at station 60 be 38 feet, of course we have 6 feet of excavation, and this is marked in our estimate sheets $+6$. If the elevation at that point should be 27 feet, 5 feet of embankment is the consequence, which is marked accordingly, -5; *plus* indicating excavation, and *minus* embankment.

The usual slope for embankment is $1\frac{1}{2}$ feet horizontal to 1 foot vertical, making an angle of about 34° with the

horizon; that for earth cut, 1 to 1, or 45°, and for rock cut $\frac{1}{4}$ to 1, or 76°. The slopes of the ground surface at each station being known, together with the breadth of the intended roadway, we are prepared to calculate the cross-sectional areas, and from them to determine the cubic yards of excavation and embankment.

To facilitate these operations the following rules were prepared. With Trautwine's common diagram, and the table of squares and square roots appended to the volume. they will be found an observable assistance.



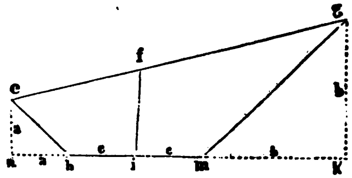
Suppose $l\,i$ the road bed, and $f\,h$ the depth of an ordi nary clay cut. Produce the side slopes until they meet in the point k. Then the angle being 45°, $ef = fk$, and ef^2 = twice the triangle efk, = the triangle ekg. For like reasons $l\,h = h\,k$, and $l\,h^2$ = triangle $l\,k\,i$. But the triangle $l\,k\,i$, taken from the triangle $e\,k\,g$, leaves the area $e\,l\,i\,g$, to find which we have therefore the following

Rule.—To half the breadth of the roadway, add the depth of the cut. From the square of this sum, subtract the square of half the breadth of the roadway for the area.

Example.—Suppose the breadth of the roadway to be 32 feet, and the depth hf, 7 feet. Half of 32 = 16, the square of which, viz., 256, becomes a constant subtrahend. 16 + 7 = 23, and a reference to the table shows the square of 23 equal to 529. Therefore 529 — 256 = 273, the area required in square feet.

If there be a regular slope, as eg (p. 58), we can plot it on the diagram, and read at once the side cuts, $g\,k$, $e\,n$. Then

the area is equal to that of the trapezoid *e n k g*, *minus* the two triangles *e n h*, *m k g*.



Calling *e n*, or its equal *n h*, *a*, *m k* or *g k*, *b*, and half the breadth of the roadway, *c*, we have the triangle *m k g* $= b \times \frac{b}{2} = \frac{b^2}{2}$. The triangle *e n h* is in like manner $= \frac{a^2}{2}$, and the area of the trapezoid $= a + b + 2\,c \times \frac{a + b}{2}$.

Then the area required $= a + b + 2\,c \times \frac{a+b}{2} - \frac{a^2 + b^2}{2}$

$$= \frac{a^2 + 2\,a\,b + 2\,a\,c + 2\,b\,c + b^2}{2} - \frac{a^2 + b^2}{2}$$

$$= \frac{2\,a\,b + 2\,a\,c + 2\,b\,c}{2} = (a + b)\,c + a.b.$$

Should the slopes be ¼ to 1, as in rock cut, the area might, in similar-wise, be shown equal to $(a + b)\,c + \frac{a.b}{4}$, and that of embankment, where the slopes are 1½ to 1, equal to $(a + b)\,c + \frac{3\,a.b}{2}$. Wherefore, we arrive at the following general rule for finding cross-sectional areas, where the ground surface is a regular declivity:

Multiply the *sum* of the side cuttings by half the breadth of the roadway, and mark the product. Multiply the *product* of the side cuttings by the ratio of the side slopes to 1, and add this result to the previous product marked for the area.

Example 1.—Earth excavation—road bed 32 feet, side

slopes 1 to 1, right cutting 12 feet, left cutting 3 feet. Required the cross-sectional area. Here $(a + b)\,c + a\,b = (12 + 3)\,16 + 12 \times 3 = 240 + 36 = 276$ square feet, the area required.

Example 2.—Rock cut—road bed 28 feet, side slopes $\frac{1}{4}$ to 1, cuttings as before. Required the area. Here $(a + b)\,c + \frac{a \times b}{4} = 210 + \frac{36}{4} = 219$ square feet, the area.

Example 3.—Embankment—roadway 27 feet, side slopes $1\frac{1}{2}$ to 1, cuttings as before. Here $(a + b)\,c + \frac{3\,(a \times b)}{2} = 202{\cdot}5 + 54 = 256{\cdot}5$ square feet, the cross-sectional area.

Other formulæ might be given for varying surface slopes, but they would involve a simple matter, and require more time for calculation than a division of the diagram area into triangles.

To find the cubic content of an excavation or embankment:—

Multiply half the sum of the two end areas by the distance between them; thus: supposing 282 and 310 to be the end areas, and 100 feet the distance, we have $\frac{(282 + 310)}{2} \times 100 = 29600$ cubic feet $= 1096{\cdot}3$ cubic yards. Or we may multiply the sum of the end areas by 100, and divide the product by 6 and 9, the factors of 54, which is the number of cubic feet contained in two cubic yards; thus, $\frac{59200}{6} = 9866{\cdot}6$, and this divided by $9 = 1096{\cdot}3$, as above.

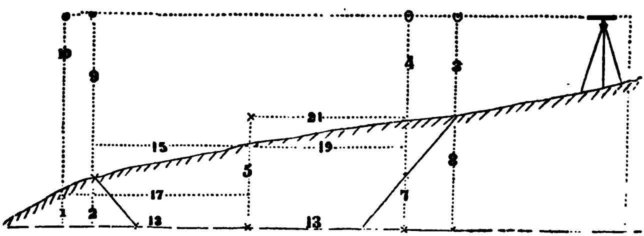
PRISMOIDAL FORMULA.

To the two end areas add four times the middle area, and multiply the sum by one-sixth of the length of the prismoid. Thus: from the foregoing example, the sum of the end areas, 592, added to four times their mean, 1184, gives 1776, and 1776 × 16·7 = 29659·2 cubic feet, = 1098·5 cubic yards. The former rule is approximate—sufficiently so for rough preliminary calculations, but where strict correctness is required, as in a final field estimate, the prismoidal formula should be used. It applies to all solid bodies with plane faces and parallel ends.

ARTICLE XXIV.

SIDE STAKING.

THE object in side staking is to find the point where the surface of the ground intersects the slope of the road



formation. For performing this work with level and rod, place the instrument in such a position as to command as many stations as possible, whether they be on the upper

or lower side, and ascertain by transfers from the bench the height of your instrument with reference to grade. This will facilitate operations by giving usually small numbers to work with. Knowing your rate of grade, subtract or add, as the case may be, as you change from station to station.

Having the level in place, we will suppose on an excavation, our object is first to place the lower side stake. Let the width of the road bed be 26 feet, and the side slopes 1 to 1. Suppose the instrument to be 11 feet above grade and the centre cut 5 feet, which latter is already recorded in the field book for construction duties. Looking at the fall of the hill we judge that at the distance of 17 feet from the centre stake the descent is 1·5 feet, which would give us 3·5 feet cutting. We take an observation at that point and find that the rod reads 10 feet, which, subtracted from our height above grade, leaves us only one foot cutting, and shows that our judgment has been at fault. The distance measured is too much, for were we to increase the distance we would reduce the cutting, and it is therefore evident that we are too far from the centre Let us then make the distance from the centre stake 15 feet, and take another sight, when the rod reads 9, showing 2 feet cutting, which, added to half the road formation, slope being 1 to 1, equals the distance measured. The stake is therefore correct. The same process applies to the upper side.

The only difference in staking out for banks is that you add 1½ times the height of the bank to the distance measured, or, as a general rule, the height of the bank multiplied by the ratio of the side slopes to unity.

Some judgment is required to be expeditious in this work, which is obtained only by experience.

The following is a common form of field record for construction:

						Left Side.		Centre.		Right Side	
Sta	Dist.	Course.	Mag. Course.	Elevation	Grade.	Dist.	A or B	A	B	Dist.	A or B
530	100	N 20°00 W	N 20·05 W	+ 560·4	+ 555·4	15·0	2·0 A	5·0		21·0	8·0 A

NATURAL SINES AND TANGENTS

(63)

′	0°	1°	2°	3°	4°	5°	6°	7°	′
0	000 0000	017 4524	034 8995	052 3360	069 7565	087 1557	104 5285	121 8693	60
1	2909	7432	035 1902	6264	070 0467	4455	8178	122 1581	59
2	5818	018 0341	4809	9169	3368	7353	105 1070	4468	58
3	8727	3249	7716	053 2074	6270	088 0251	3963	7355	57
4	001 1636	6158	036 0623	4979	9171	3148	6856	123 0241	56
5	4544	9066	3530	7883	071 2073	6046	9748	312[illegible]	55
6	7453	019 1974	6437	054 0788	4974	8943	106 2641	6015	54
7	002 0362	4883	9344	3693	7876	089 1840	5533	8901	53
8	3271	7791	037 2251	6597	072 0777	4738	8425	124 1788	52
9	6180	020 0699	5158	9502	3678	7635	107 1318	4674	51
10	9089	3608	8065	055 2406	6580	090 0532	4210	7560	50
11	003 1998	6516	038 0971	5311	9481	3429	7102	125 0446	49
12	4907	9424	3878	8215	073 2382	6326	9994	3332	48
13	7815	021 2332	6785	056 1119	5283	9223	108 2885	6218	47
14	004 0724	5241	9692	4024	8184	091 2119	5777	9104	46
15	3633	8149	039 2598	6928	074 1085	5016	8669	126 1990	45
16	6542	022 1057	5505	9832	3986	7913	109 1560	4875	44
17	9451	3965	8411	057 2736	6887	092 0809	4452	7761	43
18	005 2360	6873	040 0318	5640	9787	3706	7343	127 0646	42
19	5268	9781	4224	8544	075 2688	6602	110 0234	3531	41
20	8177	023 2690	7131	058 1448	5589	9499	3126	6416	40
21	006 1086	5598	041 0037	4352	8489	093 2395	6017	9302	39
22	3995	8506	2944	7256	076 1390	5291	8908	128 2186	38
23	6904	024 1414	5850	059 0160	4290	8187	111 1799	5071	37
24	9813	4322	8757	3064	7190	094 1083	4689	7956	36
25	007 2721	7230	042 1663	5967	077 0091	3979	7580	129 0841	35
26	5630	025 0138	4569	8871	2991	6875	112 0471	3725	34
27	8539	3046	7475	060 1775	5891	9771	3361	6609	33
28	008 1448	5954	043 0382	4678	8791	095 2[illegible]66	6252	9494	32
29	4357	8862	3288	7582	078 1691	5562	9142	130 2378	31
30	7265	026 1769	6194	061 0485	4591	8458	113 2032	5262	30
31	009 0174	4677	9100	3389	7491	096 1353	4922	8146	29
32	3083	7585	044 2006	6292	079 0391	4248	7812	131 1030	28
33	5992	027 0493	4912	9196	3290	7144	114 0702	3913	27
34	8900	3401	7818	062 2099	6190	097 0039	3592	6797	26
35	010 1809	6309	045 0724	5002	9090	2934	6482	9681	25
36	4718	9216	3630	7905	080 1989	5829	9372	132 2564	24
37	7627	028 2124	6536	063 0808	4889	8724	115 2261	5447	23
38	011 0535	5032	9442	3711	7788	098 1619	5151	8330	22
39	3444	7940	046 2347	6614	081 0687	4514	8040	133 1213	21
40	6353	029 0847	5253	9517	3587	7408	116 0929	4096	20
41	9261	3755	8159	064 2420	6486	099 0303	3818	6979	19
42	012 2170	6662	047 1065	5323	9385	3197	6707	9862	18
43	5079	9570	3970	8226	082 2284	6092	9596	134 2744	17
44	7987	030 2478	6876	065 1129	5183	8986	117 2485	5627	16
45	013 0896	5385	9781	4031	8082	100 1881	5374	8509	15
46	3805	8293	048 2687	6934	083 0981	4775	8263	135 1392	14
47	6713	031 1200	5592	9836	3880	7669	118 1151	4274	13
48	9622	4108	8498	066 2739	6778	101 0563	4040	7156	12
49	014 2530	7015	049 1403	5641	9677	3457	6928	136 0038	11
50	5439	9922	4308	8544	084 2576	6351	9816	2919	10
51	8348	032 2830	7214	067 1446	5474	9245	119 2704	5801	9
52	015 1256	5737	050 0119	4349	8373	102 2138	5593	8683	8
53	4165	8644	3024	7251	085 1271	5032	8481	137 1564	7
54	7073	033 1552	5929	068 0153	4169	7925	120 1368	4445	6
55	9982	4459	8835	3055	7067	103 0819	4256	7327	5
56	016 2890	7366	051 1740	5957	9966	3712	7144	138 0208	4
57	5799	034 0274	4645	8859	086 2864	6605	121 0031	3089	3
58	8707	3181	7550	069 1761	5762	9499	2919	5970	2
59	017 1616	6088	052 0455	4663	8660	104 2392	5806	8850	1
60	4524	8995	3360	7565	087 1557	5285	8693	139 1731	0
′	89°	88°	87°	86°	85°	84°	83°	82°	′

′	0°	1°	2°	3°	4°	5°	6°	7°	′
0	000 0000	017 4551	034 9208	052 4078	069 9268	087 4887	105 1042	122 7846	60
1	2909	7460	035 2120	6995	070 2191	7818	3983	123 0798	59
2	5818	018 0370	5033	9912	5115	088 0749	6925	3752	58
3	8727	3280	7945	053 2829	8038	3681	9866	6705	57
4	001 1636	6190	036 0858	5746	071 0961	6612	106 2808	9658	56
5	4544	9100	3771	8663	3885	9544	5750	124 2612	55
6	7453	019 2010	6683	054 1581	6809	089 2476	8692	5566	54
7	002 0362	4920	9596	4498	9733	5408	107 1634	8520	53
8	3271	7830	037 2509	7416	072 2657	8341	4576	125 1474	52
9	6180	020 0740	5422	055 0333	5581	090 1273	7519	4429	51
10	9089	3650	8335	3251	8505	4206	108 0462	7384	50
11	003 1998	6560	038 1248	6169	073 1430	7138	3405	126 0339	49
12	4907	9470	4161	9087	4354	091 0071	6348	3294	48
13	7816	021 2380	7074	056 2005	7279	3004	9291	6249	47
14	004 0725	5291	9988	4923	074 0203	5938	109 2234	9205	46
15	3634	8201	039 2901	7841	3128	8871	5178	127 2161	45
16	6542	022 1111	5814	057 0759	6053	092 1804	8122	5117	44
17	9451	4021	8728	3678	8979	4738	110 1066	8073	43
18	005 2360	6932	040 1641	6596	075 1904	7672	4010	128 1030	42
19	5269	9842	4555	9515	4829	093 0606	6955	3986	41
20	8178	023 2753	7469	058 2434	7755	3540	9899	6943	40
21	006 1087	5663	041 0383	5352	076 0680	6474	111 2844	9900	39
22	3996	8574	3296	8271	3606	9409	5789	129 2858	38
23	6905	024 1484	6210	059 1190	6532	094 2344	8734	5815	37
24	9814	4395	9124	4109	9458	5278	112 1680	8773	36
25	007 2723	7305	042 2038	7029	077 2384	8213	4625	130 1731	35
26	5632	025 0216	4952	9948	5311	095 1148	7571	4690	34
27	8541	3127	7866	060 2867	8237	4084	113 0517	7648	33
28	008 1450	6038	043 0781	5787	078 1164	7019	3463	131 0607	32
29	4360	8948	3695	8706	4090	9955	6410	3566	31
30	7269	026 1859	6609	061 1626	7017	096 2890	9356	6525	30
31	009 0178	4770	9524	4546	9944	5826	114 2303	9484	29
32	3087	7681	044 2438	7466	079 2871	8763	5250	132 2444	28
33	5996	027 0592	5353	062 0386	5798	097 1699	8197	5404	27
34	8905	3503	8268	3306	8726	4635	115 1144	8364	26
35	010 1814	6414	045 1183	6226	080 1653	7572	4092	133 1324	25
36	4724	9325	4097	9147	4581	098 0509	7039	4285	24
37	7633	028 2236	7012	063 2067	7509	3446	9987	7246	23
38	011 0542	5148	9927	4988	081 0437	6383	116 2936	134 0207	22
39	3451	8059	046 2842	7908	3365	9320	5884	3168	21
40	6361	029 0970	5757	064 0829	6293	099 2257	8832	6129	20
41	9270	3882	8673	3750	9221	5194	117 1781	9091	19
42	012 2179	6793	047 1588	6671	082 2150	8133	4730	135 2053	18
43	5088	9705	4503	9592	5078	100 1071	7679	5015	17
44	7998	030 2616	7419	065 2513	8007	4009	118 0628	7978	16
45	013 0907	5528	048 0334	5435	083 0936	6947	3578	136 0940	15
46	3817	8439	3250	8356	3865	9886	6528	3903	14
47	6726	031 1351	6166	066 1278	6794	101 2824	9478	6866	13
48	9635	4263	9082	4199	9723	5763	119 2428	9830	12
49	014 2545	7174	049 1997	7121	084 2653	8702	5378	137 2793	11
50	5454	032 0086	4913	067 0043	5583	102 1641	8329	5757	10
51	8364	2998	7829	2965	8512	4580	120 1279	8721	9
52	015 1273	5910	050 0746	5887	085 1442	7520	4230	138 1685	8
53	4183	8822	3662	8809	4372	103 0460	7182	4650	7
54	7093	033 1734	6578	068 1732	7302	3399	121 0133	7615	6
55	016 0002	4646	9495	4654	086 0233	6340	3085	139 0580	5
56	2912	7558	051 2411	7577	3163	9280	6036	3545	4
57	5821	034 0471	5328	069 0499	6094	104 2220	8988	6510	3
58	8731	3383	8244	3422	9025	5161	122 1941	9476	2
59	017 1641	6295	052 1161	6345	087 1956	8101	4893	140 2442	1
60	4551	9208	4078	9268	4887	105 1042	7846	5408	0
′	89°	88°	87°	86°	85°	84°	83°	82°	′

NAT. COTAN.

′	8°	9°	10°	11°	12°	13°	14°	15°	′
0	139 1731	156 4345	173 6482	190 8090	207 9117	224 9511	241 9219	258 8190	60
1	4612	7218	9346	191 0945	208 1962	225 2345	242 2041	259 1006	59
2	7492	157 0091	174 2211	3801	4807	5179	4863	3810	58
3	140 0372	2963	5075	6656	7652	8013	7685	6619	57
4	3252	5836	7939	9510	209 0497	226 0846	243 0507	9428	56
5	6132	8708	175 0803	192 2365	3341	3680	3329	260 2237	55
6	9012	158 1581	3667	5220	6186	6513	6150	5045	54
7	141 1892	4453	6531	8074	9030	9346	8971	7853	53
8	4772	7325	9395	193 0928	210 1874	227 2179	244 1792	261 0662	52
9	7651	159 0197	176 2258	3782	4718	5012	4613	3469	51
10	142 0531	3069	5121	6636	7561	7844	7433	6277	50
11	3410	5940	7984	9490	211 0405	228 0677	245 0254	9085	49
12	6289	8812	177 0847	194 2344	3248	3509	3074	262 1892	48
13	9168	160 1683	3710	5197	6091	6341	5894	4699	47
14	143 2047	4555	6573	8050	8934	9172	8713	7506	46
15	4926	7426	9435	195 0903	212 1777	229 2004	246 1533	263 0312	45
16	7805	161 0297	178 2298	3756	4619	4835	4352	3118	44
17	144 0684	3167	5160	6609	7462	7666	7171	5925	43
18	3562	6038	8022	9461	213 0304	230 0497	9990	8730	42
19	6440	8909	179 0884	196 2314	3146	3328	247 2809	264 1536	41
20	9319	162 1779	3746	5166	5988	6159	5627	4342	40
21	145 2197	4650	6607	8018	8829	6989	8445	7147	39
22	5075	7520	9469	197 0870	214 1671	231 1819	248 1263	9952	38
23	7953	163 0390	180 2330	3722	4512	4649	4081	265 2757	37
24	146 0830	3260	5191	6573	7353	7479	6899	5561	36
25	3708	6129	8052	9425	215 0194	232 0309	9716	8366	35
26	6585	8999	181 0913	198 2276	3035	3138	249 2533	266 1170	34
27	9463	164 1868	3774	5127	5876	5967	5350	3973	33
28	147 2340	4738	6635	7978	8716	8796	8167	6777	32
29	5217	7607	9495	199 0829	216 1556	233 1625	250 0984	9581	31
30	8094	165 0476	182 2355	3679	4396	4454	3800	267 2384	30
31	148 0971	3345	5215	6530	7236	7282	6616	5187	29
32	3848	6214	8075	9380	217 0076	234 0110	9432	7989	28
33	6724	9082	183 0935	200 2230	2915	2938	251 2248	268 0792	27
34	9601	166 1951	3795	5080	5754	5766	5063	3594	26
35	149 2477	4819	6654	7930	8593	8594	7879	6396	25
36	5353	7687	9514	201 0779	218 1432	235 1421	252 0694	9198	24
37	8230	167 0556	184 2373	3629	4271	4248	3508	269 2000	23
38	150 1106	3423	5232	6478	7110	7075	6323	4801	22
39	3981	6291	8091	9327	9948	9902	9137	7602	21
40	6857	9159	185 0949	202 2176	219 2786	236 2729	253 1952	270 0403	20
41	9733	168 2026	3808	5024	5624	5555	4766	3204	19
42	151 2608	4894	6666	7873	8462	8381	7579	6004	18
43	5484	7761	9524	203 0721	220 1300	237 1207	254 0393	8805	17
44	8359	169 0628	186 2382	3569	4137	4033	3206	271 1605	16
45	152 1234	3495	5240	6418	6974	6859	6019	4404	15
46	4109	6362	8098	9265	9811	9684	8832	7204	14
47	6984	9228	187 0956	204 2113	221 2648	238 2510	255 1645	272 0003	13
48	9858	170 2095	3813	4961	5485	5335	4458	2802	12
49	153 2733	4961	6670	7808	8321	8159	7270	5601	11
50	5607	7828	9528	205 0655	222 1158	239 0984	256 0082	8400	10
51	8482	171 0694	188 2385	3502	3994	3808	2894	273 1198	9
52	154 1356	3560	5241	6349	6830	6633	5705	3997	8
53	4230	6425	8098	9195	9666	9457	8517	6794	7
54	7104	9291	189 0954	206 2042	223 2501	240 2280	257 1328	9592	6
55	9978	172 2156	3811	4888	5337	5104	4139	274 2390	5
56	155 2851	5022	6667	7734	8172	7927	6950	5187	4
57	5725	7887	9523	207 0580	224 1007	241 0751	9760	7984	3
58	8598	173 0752	190 2379	3426	3842	3574	258 2570	275 0781	2
59	156 1472	3617	5234	6272	6676	6396	5381	3577	1
60	4345	6482	8090	9117	9511	9219	8190	6374	0
′	81°	80°	79°	78°	77°	76°	75°	74°	′

′	8°	9°	10°	11°	12°	13°	14°	15°	′
0	140 5408	158 3844	176 3270	194 3803	212 5566	230 8682	249 3280	267 9492	60
1	8375	6826	6269	6822	8606	231 1740	6370	268 2610	59
2	141 1342	9809	9269	9841	213 1647	4811	9460	5728	58
3	4308	159 2791	177 2269	195 2861	4688	7876	250 2551	8847	57
4	7276	5774	5270	5881	7730	232 0941	5642	269 1967	56
5	142 0243	8757	8270	8901	214 0772	4007	8734	5087	55
6	3211	160 1740	178 1271	196 1922	3814	7073	251 1826	8207	54
7	6179	4724	4273	4943	6857	233 0140	4919	270 1328	53
8	9147	7708	7274	7964	9900	3207	8012	4449	52
9	143 2115	161 0692	179 0276	197 0986	215 2944	6274	252 1106	7571	51
10	5084	3677	3279	4008	5988	9342	4200	271 0694	50
11	8053	6662	6281	7031	9032	234 2410	7294	3817	49
12	144 1022	9647	9284	198 0053	216 2077	5479	253 0389	6940	48
13	3991	162 2632	180 2287	3076	5122	8548	3484	272 0064	47
14	6961	5618	5291	6100	8167	235 1617	6580	3188	46
15	9931	8603	8295	9124	217 1213	4687	9676	6313	45
16	145 2901	163 1590	181 1299	199 2148	4259	7758	254 2773	9438	44
17	5872	4576	4303	5172	7306	236 0829	5870	273 2564	43
18	8842	7563	7308	8197	218 0353	3900	8968	5690	42
19	146 1813	164 0550	182 0313	200 1222	3400	6971	255 2066	8817	41
20	4784	3537	3319	4248	6448	237 0044	5165	274 1945	40
21	7756	6525	6324	7274	9496	3116	8264	5072	39
22	147 0727	9513	9330	201 0300	219 2544	6189	256 1363	8201	38
23	3699	165 2501	183 2337	3327	5593	9262	4463	275 1330	37
24	6672	5489	5343	6354	8643	238 2336	7564	4459	36
25	9644	8478	8350	9381	220 1692	5410	257 0664	7589	35
26	148 2617	166 1467	184 1358	202 2409	4742	8485	3766	276 0719	34
27	5590	4456	4365	5437	7793	239 1560	6868	3850	33
28	8563	7446	7373	8465	221 0844	4635	9970	6981	32
29	149 1536	167 0436	185 0382	203 1494	3895	7711	258 3073	277 0113	31
30	4510	3426	3390	4523	6947	240 0788	6176	3245	30
31	7484	6417	6399	7552	9999	3864	9280	6378	29
32	150 0458	9407	9409	204 0582	222 3051	6942	259 2384	9512	28
33	3433	168 2398	186 2418	3612	6104	241 0019	5488	278 2646	27
34	6408	5390	5428	6643	9157	3097	8593	5780	26
35	9383	8381	8439	9674	223 2211	6176	260 1699	8915	25
36	151 2358	169 1373	187 1449	205 2705	5265	9255	4805	279 2050	24
37	5333	4366	4460	5737	8319	242 2334	7911	5186	23
38	8309	7358	7471	8769	224 1374	5414	261 1018	8322	22
39	152 1285	170 0351	188 0483	206 1801	4429	8494	4126	280 1459	21
40	4262	3344	3495	4834	7485	243 1575	7234	4597	20
41	7238	6338	6507	7867	225 0541	4656	262 0342	7735	19
42	153 0215	9331	9520	207 0900	3597	7737	3451	281 0873	18
43	3192	171 2325	189 2533	3934	6654	244 0819	6560	4012	17
44	6170	5320	5546	6968	9711	3902	9670	7152	16
45	9147	8314	8559	208 0003	226 2769	6984	263 2780	282 0292	15
46	154 2125	172 1309	190 1573	3038	5827	245 0068	5891	3432	14
47	5103	4304	4587	6073	8885	3151	9002	6573	13
48	8082	7300	7602	9109	227 1944	6236	264 2114	9715	12
49	155 1061	173 0296	191 0617	209 2145	5003	9320	5226	283 2857	11
50	4040	3292	3632	5181	8063	246 2405	8339	5999	10
51	7019	6288	6648	8218	228 1123	5491	265 1452	9143	9
52	9998	9285	9664	210 1255	4184	8577	4566	284 2286	8
53	156 2978	174 2282	192 2680	4293	7244	247 1663	7680	5430	7
54	5958	5279	5696	7331	229 0306	4750	266 0794	8575	6
55	8939	8277	8713	211 0369	3367	7837	3909	285 1720	5
56	157 1919	175 1275	193 1731	3407	6429	248 0925	7025	4866	4
57	4900	4273	4749	6446	9492	4013	267 0141	8012	3
58	7881	7272	7766	9486	230 2555	7102	3257	286 1159	2
59	158 0863	176 0271	194 0784	212 2525	5618	249 0191	6374	4306	1
60	3844	3270	3803	5566	8682	3280	9492	7454	0
′	81°	80°	79°	78°	77°	76°	75°	74°	′

′	16°	17°	18°	19°	20°	21°	22°	23°	′
0	275 6374	292 3717	309 0170	325 5682	342 0201	358 3679	374 6066	390 7311	60
1	9170	6499	2936	8432	2935	6395	8763	9989	59
2	276 1965	9280	5702	326 1182	5668	9110	375 1459	391 2666	58
3	4761	293 2061	8468	3932	8400	359 1825	4156	5343	57
4	7556	4842	310 1234	6681	343 1133	4540	6852	8019	56
5	277 0352	7623	3999	9430	3865	7254	9547	392 0695	55
6	3147	294 0403	6764	327 2179	6597	9968	376 2243	3371	54
7	5941	3183	9529	4928	9329	360 2682	4938	6047	53
8	8736	5963	311 2294	7676	344 2060	5395	7632	9722	52
9	278 1530	8743	5058	328 0424	4791	8108	377 0327	393 1397	51
10	4324	295 1522	7822	3172	7521	361 0821	3021	4071	50
11	7118	4302	312 0586	5919	345 0252	3534	5714	6745	49
12	9911	7081	3349	8666	2982	6246	8408	9419	48
13	279 2704	9859	6112	329 1413	5712	8958	378 1101	394 2093	47
14	5497	296 2638	8875	4160	8441	362 1669	3794	4766	46
15	8290	5416	313 1638	6906	346 1171	4380	6486	7439	45
16	280 1083	8194	4400	9653	3900	7091	9178	395 0111	44
17	3875	297 0971	7163	320 2398	6628	9802	379 1870	2783	43
18	6667	3749	9925	5144	9357	363 2512	4562	5455	42
19	9459	6526	314 2686	7889	347 2085	5222	7253	8127	41
20	281 2251	9303	5448	331 0634	4812	7932	9944	396 0798	40
21	5042	298 2079	8209	3379	7540	364 0641	380 2634	3468	39
22	7833	4856	315 0969	6123	348 0267	3351	5324	6139	38
23	282 0624	7632	3730	8867	2994	6059	8014	8809	37
24	3415	299 0408	6490	332 1611	5720	8768	381 0704	397 1479	36
25	6205	3184	9250	4355	8447	365 1476	3393	4148	35
26	8995	5959	316 2010	7098	349 1173	4184	6082	6818	34
27	283 1785	8734	4770	9841	3898	6891	8770	9486	33
28	4575	300 1509	7529	333 2584	6624	9599	382 1459	398 2155	32
29	7364	4284	317 0288	5326	9349	366 2306	4147	4823	31
30	284 0153	7058	3047	8069	350 2074	5012	6834	7491	30
31	2942	9832	5805	334 0810	4798	7719	9522	399 0158	29
32	5731	301 2606	8563	3552	7523	367 0425	383 2209	2825	28
33	8520	5380	318 1321	6293	351 0246	3130	4895	5492	27
34	285 1308	8153	4079	9034	2970	5836	7582	8158	26
35	4096	302 0926	6836	335 1775	5963	8541	384 0268	400 0825	25
36	6884	3699	9593	4516	8416	368 1246	2953	3490	24
37	9671	6471	319 2350	7256	352 1139	3950	5639	6156	23
38	286 2458	9244	5106	9996	3862	6654	8324	8821	22
39	5246	303 2016	7863	336 2735	6584	9358	385 1008	401 1486	21
40	8032	4788	320 0619	5475	9306	369 2061	3693	4150	20
41	287 0819	7559	3374	8214	353 2027	4765	6377	6814	19
42	3605	304 0331	6130	337 0953	4748	7468	9060	9478	18
43	6391	3102	8885	3691	7469	370 0170	386 1744	402 2141	17
44	9177	5872	321 1640	6429	354 0190	2872	4427	4804	16
45	288 1963	8643	4395	9167	2910	5574	7110	7467	15
46	4748	305 1413	7149	338 1905	5630	8276	9792	403 0129	14
47	7533	4183	9903	4642	8350	371 0977	387 2474	2791	13
48	289 0318	6953	322 2657	7379	355 1070	3678	5156	5453	12
49	3103	9723	5411	339 0116	3785	6379	7837	8114	11
50	5887	306 2492	8164	2852	6508	9079	388 0518	404 077	10
51	8671	5261	323 0917	5589	9226	372 1780	3199	3436	9
52	290 1455	8030	3670	8325	356 1944	4479	5880	6096	8
53	4239	307 0798	6422	340 1060	4662	7179	8560	8756	7
54	7022	3566	9174	3796	7380	9878	389 1240	405 1416	6
55	9805	6334	324 1926	6531	357 0097	373 2577	3919	4075	5
56	291 2588	9102	4678	9265	2814	5275	6598	6734	4
57	5371	308 1869	7429	341 2000	5531	7973	9277	9393	3
58	8153	4636	325 0180	4734	8248	374 0671	390 1955	406 2051	2
59	292 0935	7403	2931	7468	358 0964	3369	4653	4709	1
60	3717	309 0170	5682	342 0201	3679	6066	7311	7366	0
′	73°	72°	71°	70°	69°	68°	67°	66°	′

NAT. COSINE.

′	16°	17°	18°	19°	20°	21°	22°	23°	′
0	286 7454	305 7307	324 9197	344 3276	363 9702	383 8640	404 0262	424 4748	60
1	287 0602	306 0488	325 2413	6530	364 2997	384 1978	3646	8182	59
2	3751	3670	5630	9785	6292	5317	7031	425 1616	58
3	6900	6852	8848	345 3040	9588	8656	405 0417	5051	57
4	288 0050	307 0034	326 2066	6296	365 2885	385 1996	3804	8487	56
5	3201	3218	5284	9553	6182	5337	7191	426 1924	55
6	6352	6402	8504	346 2810	9480	8679	406 0579	5361	54
7	9503	9586	327 1724	6068	366 2779	386 2021	3968	8800	53
8	289 2655	308 2771	4944	9327	6079	5364	7358	427 2239	52
9	5808	5957	8165	347 2586	9379	8708	407 0748	5680	51
10	8961	9143	328 1387	5846	367 2680	387 2053	4139	9121	50
11	290 2114	309 2330	4610	9107	5981	5398	7531	428 2563	49
12	5269	5517	7833	348 2368	9284	8744	408 0924	6005	48
13	8423	8705	329 1056	5630	368 2587	388 2091	4318	9449	47
14	291 1578	310 1893	4281	8893	5890	5439	7713	429 2894	46
15	4734	5083	7505	349 2156	9195	8787	409 1108	6339	45
16	7890	8272	330 0731	5420	369 2500	389 2136	4504	9785	44
17	292 1047	311 1462	3957	8685	5806	5486	7901	430 3232	43
18	4205	4653	7184	350 1950	9112	8837	410 1299	6680	42
19	7363	7845	331 0411	5216	370 2420	390 2189	4697	431 0129	41
20	293 0521	312 1036	3639	8483	5728	5541	8097	3579	40
21	3680	4229	6868	351 1750	9036	8894	411 1497	7030	39
22	6839	7422	332 0097	5018	371 2346	391 2247	4898	432 0481	38
23	9999	313 0616	3327	8287	5656	5602	8300	3933	37
24	294 3160	3810	6557	352 1556	8967	8957	412 1703	7386	36
25	6321	7005	9788	4826	372 2278	392 2313	5106	433 0840	35
26	9483	314 0200	333 3020	8096	5590	5670	8510	4295	34
27	295 2645	3396	6252	353 1368	8903	9027	413 1915	7751	33
28	5808	6593	9485	4640	373 2217	393 2386	5321	434 1208	32
29	8971	9790	334 2719	7912	5532	5745	8728	4665	31
30	296 2135	315 2988	5953	354 1186	8847	9105	414 2136	8124	30
31	5299	6186	9188	4460	374 2163	394 2465	5544	435 1583	29
32	8464	9585	335 2424	7734	5479	5827	8953	5043	28
33	297 1630	316 2585	5660	355 1010	8797	9189	415 2363	8504	27
34	4796	5785	8896	4286	375 2115	395 2552	5774	436 1966	26
35	7962	8986	336 2134	7562	5433	5916	9186	5429	25
36	298 1129	317 2187	5372	356 0840	8753	9280	416 2598	8893	24
37	4297	5389	8610	4118	376 2073	396 2645	6012	437 2357	23
38	7465	8591	337 1850	7397	5394	6011	9426	5823	22
39	299 0634	318 1794	5090	357 0676	8716	9378	417 2841	9289	21
40	3803	4998	8330	3956	377 2038	397 2746	6257	438 2756	20
41	6973	8202	338 1570	7237	5361	6114	9673	6224	19
42	300 0144	319 1407	4815	358 0518	8685	9483	418 3091	9693	18
43	3315	4613	8056	3801	378 2010	398 2853	6509	439 3163	17
44	6486	7819	339 1299	7083	5335	6224	9928	6634	16
45	9658	320 1025	4543	359 0367	8661	9595	419 3348	440 0105	15
46	301 2831	4232	7787	3651	379 1988	399 2968	6769	3578	14
47	6004	7440	340 1032	6936	5315	6341	420 0190	7051	13
48	9178	321 0649	4278	360 0222	8644	9715	3613	441 0526	12
49	302 2352	3858	7524	3508	380 1973	400 3089	7036	4001	11
50	5527	7067	341 0771	6795	5302	6465	421 0460	7477	10
51	8703	322 0278	4019	361 0082	8633	9841	3885	442 0954	9
52	303 1879	3489	7267	3371	381 1964	401 3218	7311	4432	8
53	5055	6700	342 0516	6660	5296	6596	422 0738	7910	7
54	8232	9912	3765	9949	8629	9974	4165	443 1390	6
55	304 1410	323 3125	7015	362 3240	382 1962	402 3354	7594	4871	5
56	4588	6338	343 0266	6531	5296	6734	423 1023	8352	4
57	7767	9552	3518	9823	8631	403 0115	4453	444 1834	3
58	305 0946	324 2766	6770	363 3115	383 1967	3496	7884	5318	2
59	4126	5981	344 0023	6408	5303	6879	424 1316	8802	1
60	7307	9197	3276	9702	8640	404 0262	4748	445 2287	0
′	73°	72°	71°	70°	69°	68°	67°	66°	′

′	24°	25°	26°	27°	28°	29°	30°	31°	′
0	406 7366	422 6183	438 3711	453 9905	469 4716	484 8096	500 0000	515 0381	60
1	407 0024	8819	6326	454 2497	7284	485 0640	2519	2874	59
2	2681	423 1455	8940	5088	9852	3184	5037	5367	58
3	5337	4090	439 1553	7679	470 2419	5727	7556	7859	57
4	7993	6725	4166	455 0269	4986	8270	501 0075	516 0351	56
5	408 0649	9360	6779	2859	7553	486 0812	2591	2842	55
6	3305	424 1994	9392	5449	471 0119	3354	5107	5333	54
7	5960	4628	440 2004	8038	2685	5895	7624	7824	53
8	8615	7262	4615	456 0627	5250	8436	502 0140	517 0314	52
9	409 1269	9895	7227	3216	7815	487 0977	2655	2804	51
10	3923	425 2528	9838	5804	472 0380	3517	5170	5293	50
11	6577	5161	441 2448	8392	2944	6057	7685	7782	49
12	9230	7793	5059	457 0979	5508	8597	503 0199	518 0270	48
13	410 1883	426 0425	7668	3566	8071	488 1136	2713	2758	47
14	4536	3056	442 0278	6153	473 0634	3674	5227	5246	46
15	7189	5687	2887	8739	3197	6212	7740	7733	45
16	9841	8318	5496	458 1325	5759	8750	504 0252	519 0219	44
17	411 2492	427 0949	8104	3910	8321	489 1288	2765	2705	43
18	5144	3579	443 0712	6496	474 0882	3825	5276	5191	42
19	7795	6208	3319	9080	3443	6361	7788	7676	41
20	412 0445	8838	5927	459 1665	6004	8897	505 0298	520 0161	40
21	3096	428 1467	8534	4248	8564	490 1433	2809	2646	39
22	5745	4095	444 1140	6832	475 1124	3968	5319	5130	38
23	8395	6723	3746	9415	3683	6503	7828	7613	37
24	413 1044	9351	6352	460 1998	6242	9039	506 0338	521 0096	36
25	3693	429 1979	8957	4580	8801	491 1572	2846	2579	35
26	6342	4606	445 1562	7162	476 1359	4105	5355	5061	34
27	8990	7233	4167	9744	3917	6639	7863	7543	33
28	414 1638	9859	6771	461 2325	6474	9171	507 0370	522 0024	32
29	4285	430 2485	9375	4906	9031	492 1704	2877	2505	31
30	6932	5111	446 1978	7486	477 1588	4236	5384	4986	30
31	9579	7736	4581	462 0066	4144	6767	7890	7466	29
32	415 2226	431 0361	7184	2646	6700	9298	508 0396	9945	28
33	4872	2986	9786	5225	9255	493 1829	2901	523 2424	27
34	7517	5610	447 2388	7804	478 1810	4359	5406	4903	26
35	416 0163	8234	4990	463 0382	4364	6889	7910	7381	25
36	2808	432 0857	7591	2960	6919	9419	509 0414	9859	24
37	5453	3481	448 0192	5538	9472	494 1948	2918	524 2336	23
38	8097	6103	2792	8115	479 2026	4476	5421	4813	22
39	417 0741	8726	5392	464 0692	4579	7005	7924	7290	21
40	3385	433 1348	7992	3269	7131	9532	510 0426	9766	20
41	6028	3970	449 0591	5845	9683	495 2060	2928	525 2241	19
42	8671	6591	3190	8420	480 2235	4587	5429	4717	18
43	418 1313	9212	5789	465 0996	4786	7113	7930	7191	17
44	3956	434 1832	8387	3571	7337	9639	511 0431	9665	16
45	6597	4453	450 0984	6145	9888	496 2165	2931	526 2139	15
46	9239	7072	3582	8719	481 2438	4690	5431	4613	14
47	419 1880	9692	6179	466 1293	4987	7215	7930	7085	13
48	4521	435 2311	8775	3866	7537	9740	512 0429	9558	12
49	7161	4930	451 1372	6439	482 0086	497 2264	2927	527 2030	11
50	9801	7548	3967	9012	2634	4787	5425	4502	10
51	420 2441	436 0166	6563	467 1584	5182	7310	7923	6973	9
52	5080	2784	9158	4156	7730	9833	513 0420	9443	8
53	7719	5401	452 1753	6727	483 0277	498 2355	2916	528 1914	7
54	421 0358	8018	4347	9298	2824	4877	5413	4383	6
55	2996	437 0634	6941	468 1869	5370	7399	7908	6853	5
56	5634	3251	9535	4439	7916	9920	514 0404	9322	4
57	8272	5866	453 2128	7009	484 0462	499 2441	2899	529 1790	3
58	422 0909	8482	4721	9578	3007	4961	5393	4258	2
59	3546	438 1097	7313	469 2147	5552	7481	7887	6726	1
60	6183	3711	9905	4716	8096	500 0000	515 0381	9193	0
′	65°	64°	63°	62°	61°	60°	59°	58°	′

NAT. COSINE.

′	24°	25°	26°	27°	28°	29°	30°	31°	′
0	445 2287	466 3077	487 7326	509 5254	531 7094	554 3091	577 3503	600 8606	60
1	5773	6618	488 0927	8919	532 0826	6894	7382	601 2566	59
2	9260	467 0161	4530	510 2585	4559	555 0698	578 1262	6527	58
3	446 2747	3705	8133	6252	8293	4504	5144	602 0490	57
4	6236	7250	489 1737	9919	533 2029	8311	9027	4454	56
5	9726	468 0796	5343	511 3588	5765	556 2119	579 2912	8419	55
6	447 3216	4342	8949	7259	9503	5929	6797	603 2386	54
7	6708	7890	490 2557	512 0930	534 3242	9739	590 0684	6354	53
8	448 0200	469 1439	6166	4602	6981	557 3551	4573	604 0323	52
9	3693	4988	9775	8275	535 0723	7364	8462	4294	51
10	7187	8539	491 3386	513 1950	4465	558 1179	581 2353	8260	50
11	449 0682	470 2090	6997	5625	8208	4994	6245	605 2240	49
12	4178	5643	492 0610	9302	536 1953	8811	582 0139	6215	48
13	7675	9196	4224	514 2980	5699	559 2629	4034	606 0192	47
14	450 1173	471 2751	7838	6658	9446	6449	7930	4170	46
15	4672	6306	493 1454	515 0338	537 3194	560 0269	563 1828	8149	45
16	8171	9863	5071	4019	6943	4091	5726	607 2130	44
17	451 1672	472 3420	8689	7702	538 0694	7914	9627	6112	43
18	5173	6978	494 2308	516 1385	4445	561 1738	584 3528	608 0095	42
19	8676	473 0538	5928	5069	8198	5564	7431	4080	41
20	452 2179	4098	9549	8755	539 1952	9391	585 1335	8067	40
21	5683	7659	495 3171	517 2441	5707	562 3219	5241	609 2054	39
22	9188	474 1222	6794	6129	9464	7048	9148	6043	38
23	453 2694	4785	496 0418	9818	540 3221	563 0879	586 3056	610 0034	37
24	6201	8349	4043	518 3508	6980	4710	6965	4026	36
25	9709	475 1914	7669	7199	541 0740	8543	587 0876	8019	35
26	454 3218	5481	497 1297	519 0891	4501	564 2378	4788	611 2014	34
27	6728	9048	4925	4584	8263	6213	8702	6011	33
28	455 0238	476 2616	8554	8278	542 2027	565 0050	588 2616	612 0008	32
29	3750	6185	498 2185	520 1974	5791	3888	6533	4007	31
30	7263	9755	5816	5671	9557	7728	589 0450	8008	30
31	456 0776	477 3326	9449	9368	543 3324	566 1568	4369	613 2010	29
32	4290	6899	499 3082	521 3067	7092	5410	8289	6013	28
33	7806	478 0472	6717	6767	544 0862	9254	590 2211	614 0018	27
34	457 1322	4046	500 0352	522 0468	4632	567 3098	6134	4024	26
35	4839	7621	3989	4170	8404	6944	591 0058	8032	25
36	8357	479 1197	7627	7874	545 2177	568 0791	3984	615 2041	24
37	458 1877	4774	501 1266	523 1578	5951	4639	7910	6052	23
38	5397	8352	4906	5284	9727	8488	592 1839	616 0064	22
39	8918	480 1932	8547	8990	546 3503	569 2339	5768	4077	21
40	459 2439	5512	502 2189	524 2698	7281	6191	9699	8092	20
41	5962	9093	5832	6407	547 1060	570 0045	593 3632	617 2108	19
42	9486	481 2675	9476	525 0117	4840	3899	7565	6126	18
43	460 3011	6258	503 3121	3829	8621	7755	594 1501	618 0145	17
44	6537	9842	6768	7541	548 2404	571 1612	5437	4166	16
45	461 0063	482 3427	504 0415	526 1255	6188	5471	9375	8188	15
46	3591	7014	4063	4969	9973	9331	595 3314	619 2211	14
47	7119	483 0601	7713	8685	549 3759	572 3192	7255	6236	13
48	462 0649	4189	505 1363	527 2402	7547	7054	596 1196	620 0263	12
49	4179	7778	5015	6120	550 1335	573 0918	5140	4291	11
50	7710	484 1368	8668	9839	5125	4783	9084	8320	10
51	463 1243	4959	506 2322	528 3560	8916	8649	597 3030	621 2351	9
52	4776	8552	5977	7281	551 2708	574 2516	6978	6383	8
53	8310	485 2145	9633	529 1004	6502	6385	598 0926	622 0417	7
54	464 1845	5739	507 3290	4727	552 0297	575 0255	4877	4452	6
55	5382	9334	6948	8452	4093	4126	8828	8488	5
56	8919	486 2931	508 0607	530 2178	7890	7999	599 2781	623 2527	4
57	465 2457	6528	4267	5906	553 1688	576 1873	6735	6566	3
58	5996	487 0126	7929	9634	5488	5748	600 0691	624 0607	2
59	9536	3726	509 1591	531 3364	9298	9625	4648	4650	1
60	466 3077	7326	5254	7094	554 3091	577 3503	8606	8694	0
′	65°	64°	63°	62°	61°	60°	59°	58°	′

′	32°	33°	34°	35°	36°	37°	38°	39°	′
0	529 9193	544 6390	559 1929	573 5764	587 7853	601 8150	615 6615	629 3204	60
1	530 1659	8830	4340	8147	588 0206	602 0473	8907	5464	59
2	4125	545 1269	6751	574 0529	2558	2795	616 1198	7724	58
3	6591	3707	9162	2911	4910	5117	3489	9983	57
4	9057	6145	560 1572	5292	7262	7439	5780	630 2242	56
5	531 1521	8583	3981	7672	9613	9760	8069	4500	55
6	3986	546 1020	6390	575 0053	589 1964	603 2080	617 0359	6758	54
7	6450	3456	8798	2432	4314	4400	2648	9015	53
8	8913	5892	561 1206	4811	6663	6719	4936	631 1272	52
9	532 1376	8328	3614	7190	9012	9038	7224	3528	51
10	3839	547 0763	6021	9568	590 1361	604 1356	9511	5784	50
11	6301	3198	8428	576 1946	3709	3674	618 1798	8039	49
12	8763	5632	562 0834	4323	6057	5991	4084	632 0293	48
13	533 1224	8066	3239	6700	8404	8308	6370	2547	47
14	3685	548 0499	5645	9076	591 0750	605 0624	8655	4800	46
15	6145	2932	8049	577 1452	3096	2940	619 0939	7053	45
16	8605	5365	563 0453	3827	5442	5255	3224	9306	44
17	534 1065	7797	2857	6202	7787	7570	5507	633 1557	43
18	3523	549 0228	5260	8576	592 0132	9884	7790	3809	42
19	5982	2659	7663	578 0950	2476	606 2198	620 0073	6059	41
20	8440	5090	564 0066	3323	4819	4511	2355	8310	40
21	535 0898	7520	2467	5696	7163	6824	4636	634 0559	39
22	3355	9950	4869	8069	9505	9136	6917	2808	38
23	5812	550 2379	7270	579 0440	593 1847	607 1447	9198	5057	37
24	8268	4807	9670	2812	4189	3758	621 1478	7305	36
25	536 0724	7236	565 2070	5183	6530	6069	3757	9553	35
26	3179	9663	4469	7553	8871	8379	6036	635 1800	34
27	5634	551 2091	6868	9923	594 1211	608 0689	8314	4046	33
28	8089	4518	9267	580 2292	3550	2998	622 0592	6292	32
29	537 0543	6944	566 1665	4661	5889	5306	2870	8537	31
30	2996	9370	4062	7030	8228	7614	5146	636 0782	30
31	5449	552 1795	6459	9397	595 0566	9922	7423	3026	29
32	7902	4220	8856	581 1765	2904	609 2229	9698	5270	28
33	538 0354	6645	567 1252	4132	5241	4535	623 1974	7513	27
34	2806	9069	3648	6498	7577	6841	4248	9756	26
35	5257	553 1492	6043	8864	9913	9147	6522	637 1998	25
36	7708	3915	8437	582 1230	596 2249	610 1452	8796	4240	24
37	539 0158	6338	568 0832	3595	4584	3756	624 1069	6481	23
38	2608	8760	3225	5959	6918	6060	3342	8721	22
39	5058	554 1182	5619	8323	9252	8363	5614	638 0961	21
40	7507	3603	8011	583 0687	597 1586	611 0666	7885	3201	20
41	9955	6024	569 0403	3050	3919	2969	625 0156	5440	19
42	540 2403	8444	2795	5412	6251	5270	2427	7678	18
43	4851	555 0864	5187	7774	8583	7572	4696	9916	17
44	7298	3283	7577	584 0136	598 0915	9873	6966	639 2153	16
45	9745	5702	9968	2497	3246	612 2173	9235	4390	15
46	541 2191	8121	570 2357	4857	5577	4473	626 1503	6626	14
47	4637	556 0539	4747	7217	7906	6772	3771	8862	13
48	7082	2956	7136	9577	599 0236	9071	6038	640 1097	12
49	9527	5373	9524	585 1936	2565	613 1369	8305	3332	11
50	542 1971	7790	571 1912	4294	4893	3666	627 0571	5566	10
51	4415	557 0206	4299	6652	7221	5964	2837	7799	9
52	6859	2621	6686	9010	9549	8260	5102	641 0032	8
53	9302	5036	9073	586 1367	600 1876	614 0556	7366	2264	7
54	543 1744	7451	572 1459	3724	4202	2852	9631	4496	6
55	4187	9865	3844	6080	6528	5147	628 1894	6728	5
56	6628	558 2279	6229	8435	8854	7442	4157	8958	4
57	9069	4692	8614	587 0790	601 1179	9736	6420	642 1189	3
58	544 1510	7105	573 0998	3145	3503	615 2029	8682	3418	2
59	3951	9517	3381	5499	5827	4322	629 0943	5647	1
60	6390	559 1929	5764	7853	8150	6615	3204	7876	0
′	57°	56°	55°	54°	53°	52°	51°	50°	′

′	32°	33°	34°	35°	36°	37°	38°	39°	′
0	624 8694	649 4076	674 5085	700 2075	726 5425	753 5541	781 2856	809 7840	60
1	625 2739	8212	9318	6411	9871	754 0102	7542	810 2658	59
2	6786	650 2350	675 3553	701 0749	727 4318	4666	782 2229	7478	58
3	626 0834	6490	7790	5089	8767	9232	6919	811 2300	57
4	4884	651 0631	676 2028	9430	728 3218	755 3799	783 1611	7124	56
5	8935	4774	6268	702 3773	7671	8369	6305	812 1951	55
6	627 2988	8918	677 0509	8118	729 2125	756 2941	784 1002	6780	54
7	7042	652 3064	4752	703 2464	6582	7514	5700	813 1611	53
8	628 1098	7211	8997	6813	730 1041	757 2090	785 0400	6444	52
9	5155	653 1360	678 3243	704 1163	5501	6668	5103	814 1280	51
10	9214	5511	7492	5515	9963	758 1248	9808	6118	50
11	629 3274	9663	679 1741	9869	731 4428	5829	786 4515	815 0958	49
12	7336	654 3817	5993	705 4224	8894	759 0413	9224	5801	48
13	630 1399	7972	680 0246	8581	732 3362	4999	787 3935	816 0646	47
14	5464	655 2129	4501	706 2940	7832	9587	8649	5493	46
15	9530	6287	8758	7301	733 2303	760 4177	788 3364	817 0343	45
16	631 3598	656 0447	681 3016	707 1664	6777	8769	8082	5195	44
17	7667	4609	7276	6028	734 1253	761 3363	789 2802	818 0049	43
18	632 1738	8772	682 1537	708 0395	5730	7959	7524	4905	42
19	5810	657 2937	5801	4763	735 0210	762 2557	790 2248	9764	41
20	9883	7103	683 0066	9133	4691	7157	6975	819 4625	40
21	633 3959	658 1271	4333	709 3504	9174	763 1759	791 1703	9488	39
22	8035	5441	8601	7878	736 3660	6363	6434	820 4354	38
23	634 2113	9612	684 2871	710 2253	8147	764 0969	792 1167	9222	37
24	6193	659 3785	7143	6630	737 2636	5577	5902	821 4093	36
25	635 0274	7960	685 1416	711 1009	7127	765 0188	793 0640	8965	35
26	4357	660 2136	5692	5390	738 1620	4800	5379	822 3840	34
27	8441	6313	9969	9772	6115	9414	794 0121	8718	33
28	636 2527	661 0492	686 4247	712 4157	739 0611	766 4031	4865	823 3597	32
29	6614	4673	8528	8543	5110	8649	9611	8479	31
30	637 0703	8856	687 2810	713 2931	9611	767 3270	795 4359	824 3364	30
31	4793	662 3040	7093	7320	740 4113	7893	9110	8251	29
32	8885	7225	688 1379	714 1712	8618	768 2517	796 3862	825 3140	28
33	638 2978	663 1413	5666	6106	741 3124	7144	8617	8031	27
34	7073	5601	9955	715 0501	7633	769 1773	797 3374	826 2925	26
35	639 1169	9792	689 4246	4898	742 2143	6404	8134	7821	25
36	5267	664 3984	8538	9297	6655	770 1037	798 2895	827 2719	24
37	9366	8178	690 2632	716 3698	743 1170	5672	7659	7620	23
38	640 3467	665 2373	7128	8100	5686	771 0309	799 2425	828 2523	22
39	7569	6570	691 1425	717 2505	744 0204	4948	7193	7429	21
40	641 1673	666 0769	5725	6911	4724	9589	800 1963	829 2337	20
41	5779	4969	692 0026	718 1319	9246	772 4233	6736	7247	19
42	9886	9171	4328	5729	745 3770	8878	801 1511	830 2160	18
43	642 3994	667 3374	8633	719 0141	8296	773 3526	6288	7075	17
44	8105	7580	693 2939	4554	746 2824	8176	802 1067	831 1992	16
45	643 2216	668 1786	7247	8970	7354	774 2827	5849	6912	15
46	6329	5995	694 1557	720 3387	747 1886	7481	803 0632	832 1834	14
47	644 0444	669 0205	5868	7806	6420	775 2137	5418	6759	13
48	4560	4417	695 0181	721 2227	748 0956	6795	804 0206	833 1686	12
49	8678	8630	4496	6650	5494	776 1455	4997	6615	11
50	645 2797	670 2845	8813	722 1075	749 0033	6118	9790	834 1547	10
51	6918	7061	696 3131	5502	4575	777 0782	805 4584	6481	9
52	646 1041	671 1280	7451	9930	9119	5448	9382	835 1418	8
53	5165	5500	697 1773	723 4361	750 3665	778 0117	806 4181	6357	7
54	9290	9721	6097	8793	8212	4788	8983	836 1298	6
55	647 3417	672 3944	698 0422	724 3227	751 2762	9460	807 3787	6242	5
56	7546	8169	4749	7663	7314	779 4135	8593	837 1188	4
57	648 1676	673 2396	9078	725 2101	752 1867	8812	808 3401	6136	3
58	5808	6624	699 3409	6540	6423	780 3492	8212	838 1087	2
59	9941	674 0854	7741	726 0982	753 0981	8173	809 3025	6041	1
60	649 4076	5085	700 2075	5425	5541	781 2856	7840	839 0996	0
′	57°	56°	55°	54°	53°	52°	51°	50°	′

NAT. SINE.

′	40°	41°	42°	43°	44°	45°	46°	47°	′
0	642 7876	656 0590	669 1306	681 9984	694 6584	707 1068	719 3398	731 3537	60
1	643 0104	2785	3468	682 2111	8676	3124	5418	552.	59
2	2332	4980	5628	4237	695 0767	5180	7438	7503	58
3	4559	7174	7789	6363	2858	7236	9457	9485	57
4	6785	9367	9948	8489	4949	9291	720 1476	732 1467	56
5	9011	657 1560	670 2108	683 0613	7039	708 1345	3494	3449	55
6	644 1236	3752	4266	2738	9128	3398	5511	5429	54
7	3461	5944	6424	4861	696 1217	5451	7528	7409	53
8	5685	8135	8582	6984	3305	7504	9544	9388	52
9	7909	658 0326	671 0739	9107	5392	9556	721 1559	733 1367	51
10	645 0132	2516	2895	684 1229	7479	709 1607	3574	3345	50
11	2355	4706	5051	3350	9565	3657	5589	5322	49
12	4577	6895	7206	5471	697 1651	5707	7602	7299	48
13	6798	9083	9361	7591	3736	7757	9615	9275	47
14	9019	659 1271	672 1515	9711	5821	9806	722 1628	734 1250	46
15	646 1240	3458	3668	685 1830	7905	710 1854	3640	3225	45
16	3460	5645	5821	3948	9988	3901	5651	5199	44
17	5679	7831	7973	6066	698 2071	5948	7661	7173	43
18	7898	660 0017	673 0125	8184	4153	7995	9671	9146	42
19	647 0116	2202	2276	686 0300	6234	711 0041	723 1681	735 1118	41
20	2334	4386	4427	2416	8315	2086	3690	3090	40
21	4551	6570	6577	4532	699 0396	4130	5698	5061	39
22	6767	8754	8727	6647	2476	6174	7705	7032	38
23	8984	661 0936	674 0876	8761	4555	8218	9712	9002	37
24	648 1199	3119	3024	687 0875	6633	712 0260	724 1719	736 0971	36
25	3414	5300	5172	2988	8711	2303	3724	2940	35
26	5628	7482	7319	5101	700 0789	4344	5729	4908	34
27	7842	9662	9466	7213	2866	6385	7734	6875	33
28	649 0056	662 1842	675 1612	9325	4942	8426	9738	8842	32
29	2268	4022	3757	688 1435	7018	713 0465	725 1741	737 0808	31
30	4480	6200	5902	3546	9093	2504	3744	2773	30
31	6692	8379	8046	5655	701 1167	4543	5746	4738	29
32	8903	663 0557	676 0190	7765	3241	6581	7747	6703	28
33	650 1114	2734	2333	9873	5314	8618	9748	8666	27
34	3324	4910	4476	689 1981	7387	714 0655	726 1748	738 0629	26
35	5533	7087	6618	4089	9459	2691	3748	2592	25
36	7742	9262	8760	6195	702 1531	4727	5747	4553	24
37	9951	664 1437	677 0901	8302	3601	6762	7745	6515	23
38	651 2158	3612	3041	690 0407	5672	8796	9743	8475	22
39	4366	5786	5181	2512	7741	715 0830	727 1740	739 0435	21
40	6572	7959	7320	4617	9811	2863	3736	2394	20
41	8778	665 0131	9459	6721	703 1879	4895	5732	4353	19
42	652 0984	2304	678 1597	8824	3947	6927	7728	6311	18
43	3189	4475	3734	691 0927	6014	8959	9722	8268	17
44	5394	6646	5871	3029	8081	716 0989	728 1716	740 0225	16
45	7598	8817	8007	5131	704 0147	3019	3710	2181	15
46	9801	666 0987	679 0143	7232	2213	5049	5703	4137	14
47	653 2004	3156	2278	9332	4278	7078	7695	6092	13
48	4206	5325	4413	692 1432	6342	9106	9686	8046	12
49	6408	7493	6547	3531	8406	717 1134	729 1677	741 0000	11
50	8609	9661	8681	5630	705 0469	3161	3668	1953	10
51	654 0810	667 1828	680 0813	7728	2532	5187	5657	3905	9
52	3010	3994	2946	9825	4594	7213	7646	5857	8
53	5209	6160	5078	693 1922	6655	9238	9635	7808	7
54	7408	8326	7209	4018	8716	718 1263	730 1623	9758	6
55	9607	668 0490	9339	6114	706 0776	3287	3610	742 1708	5
56	655 1804	2655	681 1469	8209	2835	5310	5597	3658	4
57	4002	4818	3599	694 0304	4894	7333	7583	5606	3
58	6198	6981	5728	2398	6953	9355	9568	7554	2
59	8395	9144	7856	4491	9011	719 1377	731 1553	9502	1
60	656 0590	669 1306	9984	6584	707 1068	3398	3537	743 1448	0
′	49°	48°	47°	46°	45°	44°	43°	42°	′

NAT. COSINE.

′	40°	41°	42°	43°	44°	45°	46°	47°	′
0	8390996	8692867	9004040	9325151	9656888	1·0000000	1·0355303	1·0723687	60
1	5955	7976	9309	9330591	9662511	05819	61333	29943	59
2	8400915	8703087	9014580	6034	8137	11642	67367	36203	58
3	5878	8200	9854	9341479	9673767	17469	73404	42467	57
4	8410844	8713316	9025131	6928	9399	23298	79445	48734	56
5	5812	8435	9030411	9352380	9685035	29131	85489	55006	55
6	8420782	8723556	5693	7834	9690674	34968	91538	61282	54
7	5755	8680	9040979	9363292	6316	40807	97589	67561	53
8	8430730	8733806	6267	8753	9701962	46651	1·0403645	73845	52
9	5708	8935	9051557	9374216	7610	52497	09704	80132	51
10	8440688	8744067	6851	9683	9713262	58348	15767	86423	50
11	5670	9201	9062147	9385153	8917	64201	21833	92718	49
12	8450655	8754338	7446	9390625	9724575	70058	27904	99018	48
13	5643	9478	9072748	6101	9730236	75918	33977	1·0805321	47
14	8460633	8764620	8053	9401579	5901	81782	40055	11628	46
15	5625	9765	9083360	7061	9741569	87649	46136	17939	45
16	8470620	8774912	8671	9412545	7240	93520	52221	24254	44
17	5617	8780062	9093984	8033	9752914	99394	58310	30573	43
18	8480617	5215	9300	9423523	8591	1·0105272	64402	36896	42
19	5619	8790370	9104919	9017	9764272	11153	70498	43223	41
20	8490624	5528	9940	9434513	9956	17038	76598	49554	40
21	5631	8800688	9115265	9440013	9775643	22925	82702	55889	39
22	8500640	5852	9120592	5516	9781333	28817	88809	62228	38
23	5653	8811017	5922	9451021	7027	34712	94920	68571	37
24	8510667	6186	9131255	6530	9792724	40610	1·0501034	74918	36
25	5684	8821357	6591	9462042	8424	46512	07153	81269	35
26	8520704	6531	9141929	7556	9804127	52418	13275	87624	34
27	5726	8831707	7270	9473074	9833	58326	19401	93984	33
28	8530750	6886	9152615	8595	9815543	64239	25531	1·0900347	32
29	5777	8842068	7962	9484119	9821256	70155	31664	06714	31
30	8540807	7253	9163312	9646	6973	76074	37801	13085	30
31	5839	8852440	8665	9495176	9832692	81997	43942	19460	29
32	8550873	7630	9174020	9500709	8415	87923	50087	25840	28
33	5910	8862822	9379	6245	9844141	93853	56235	32223	27
34	8560950	8017	9184740	9511784	9871	99786	62388	38610	26
35	5992	8873215	9190104	7326	9855603	1·0205723	68544	45002	25
36	8571037	8415	5471	9522871	9861339	11664	74704	51397	24
37	6084	8883619	9200841	8420	7079	17608	80867	57797	23
38	8581133	8825	6214	9533971	9872821	23555	87035	64201	22
39	6185	8894033	9211590	9526	8567	29506	93206	70609	21
40	8591240	9244	6969	9545083	9884316	35461	99381	77020	20
41	6297	8904458	9222350	9550644	9890069	41419	1·0605560	83436	19
42	8601357	9675	7734	6208	5825	47381	11742	89857	18
43	6419	8914894	9233122	9561774	9901584	53346	17929	96281	17
44	8611484	8920116	8512	7344	7346	59315	24119	1·1002709	16
45	6551	5341	9243905	9572917	9913112	65287	30313	09141	15
46	8621621	8930569	9301	8494	8881	71263	36511	15578	14
47	6694	5799	9254700	9584073	9924654	77243	42713	22019	13
48	8631768	8941032	9260102	9655	9930429	83226	48918	28463	12
49	6846	6268	5506	9595241	6208	89212	55128	34912	11
50	8641926	8951506	9270914	9600829	9941991	95203	61341	41365	10
51	7009	6747	6324	6421	7777	1·0301196	67558	47823	9
52	8652094	8961991	9281738	9612016	9953566	07194	73779	54284	8
53	7181	7238	7154	7614	9358	13195	80004	60750	7
54	8662272	8972487	9292573	9623215	9965154	19199	86233	67219	6
55	7365	7739	7996	8819	9970953	25208	92466	73693	5
56	8672460	8982994	9303421	9634427	6756	31220	98702	80171	4
57	7558	8251	8849	9640037	9982562	37235	1·0704943	86653	3
58	8682659	8993512	9314280	5651	8371	43254	11187	93140	2
59	7762	8775	9714	9651268	9994184	49277	17435	99630	1
60	869 867	9004040	9325151	6888	1·0000000	55303	23687	1·1106125	0
′	49°	48°	47°	46°	45°	44°	43°	42°	

′	48°	49°	50°	51°	52°	53°	54°	55°	′
0	743 1448	754 7096	766 0444	777 1460	788 0108	798 6355	809 0170	819 1520	60
1	3394	9004	2314	3290	1898	8105	1879	3189	59
2	5340	755 0911	4183	5120	3688	9855	3588	4856	58
3	7285	2818	6051	6949	5477	799 1604	5296	6523	57
4	9229	4724	7918	8777	7266	3352	7004	8189	56
5	744 1173	6630	9785	778 0604	9054	5100	8710	9854	55
6	3115	8535	767 1652	2431	789 0841	6847	810 0416	820 1519	54
7	5058	756 0439	3517	4258	2627	8593	2122	3183	53
8	6999	2342	5382	6084	4413	800 0338	3826	4846	52
9	8941	4246	7246	7909	6198	2083	5530	6509	51
10	745 0881	6148	9110	9733	7983	3827	7234	8170	50
11	2821	8050	768 0973	779 1557	9767	5571	8936	9832	49
12	4760	9951	2835	3380	790 1550	7314	811 0638	821 1492	48
13	6699	757 1851	4697	5202	3333	9056	2339	3152	47
14	8636	6751	6558	7024	5115	801 0797	4040	4811	46
15	746 0574	5650	8418	8845	6896	2538	5740	6469	45
16	2510	7548	769 0278	760 0665	8676	4278	7439	8127	44
17	4446	9446	2137	2485	791 0456	6018	9137	9784	43
18	6382	758 1343	3996	4304	2235	7756	812 0835	822 1440	42
19	8317	3240	5853	6123	4014	9495	2532	3096	41
20	747 0251	5136	7710	7940	5792	802 1232	4229	4751	40
21	2184	7031	9567	9757	7569	2969	5925	6405	39
22	4117	8926	770 1423	781 1574	9345	4705	7620	8059	38
23	6049	759 0820	3278	3390	792 1121	6440	9314	9712	37
24	7981	2713	5132	5205	2896	8175	813 1008	823 1364	36
25	9912	4606	6986	7019	4671	9909	2701	3015	35
26	748 1842	6498	8840	8833	6445	803 1642	4393	4666	34
27	3772	8389	771 0692	782 0646	8218	3375	6084	6316	33
28	5701	760 0290	2544	2459	9990	5107	7775	7965	32
29	7629	2170	4395	4270	793 1762	6838	9460	9614	31
30	9557	4060	6246	6082	3533	8569	814 1155	824 1262	30
31	749 1484	5949	8096	7892	5304	804 0299	2844	2909	29
32	3411	7837	9945	9702	7074	2028	4532	4556	28
33	5337	9724	772 1794	783 1511	8843	3756	6220	6202	27
34	7262	761 1611	3642	3320	794 0611	5484	7906	7847	26
35	9187	3497	5489	5127	2379	7211	9593	9491	25
36	750 1111	5383	7336	6935	4146	8938	815 1278	825 1135	24
37	3034	7268	9182	8741	5913	805 0664	2963	2778	23
38	4957	9152	773 1027	784 0547	7678	2389	4647	4420	22
39	6879	762 1036	2872	2352	9444	4113	6330	6062	21
40	8800	2919	4716	4157	795 1208	5837	8013	7703	20
41	751 0721	4802	6559	5961	2972	7560	9695	9343	19
42	2641	6683	8402	7764	4735	9283	816 1376	826 0983	18
43	4561	8564	774 0244	9566	6497	806 1005	3056	2622	17
44	6480	763 0445	2086	785 1368	8259	2726	4736	4260	16
45	8398	2325	3926	3169	796 0020	4446	6416	5897	15
46	752 0316	4204	5767	4970	1780	6166	8094	7534	14
47	2233	6082	7606	6770	3540	7885	9772	9170	13
48	4149	7960	9445	8569	5299	9603	817 1449	727 0806	12
49	6065	9838	775 1283	786 0367	7058	807 1321	3125	2440	11
50	7980	764 1714	3121	2165	8815	3038	4801	4074	10
51	9894	3590	4957	3963	797 0572	4754	6476	5708	9
52	753 1808	5465	6794	5759	2329	6470	8151	7340	8
53	3721	7340	8629	7555	4084	8185	9824	8972	7
54	5634	9214	776 0464	9350	5839	9899	818 1497	828 0603	6
55	7546	765 1087	2298	787 1145	7594	808 1612	3169	2234	5
56	9457	2960	4132	2939	9347	3325	4841	3864	4
57	754 1368	4832	5965	4732	798 1100	5037	6512	5493	3
58	3278	6704	7797	6524	2853	6749	8182	7121	2
59	5187	8574	9629	8316	4604	8460	9852	8749	1
60	7096	766 0444	777 1460	788 0108	6355	809 0170	819 1520	829 0376	0
′	41°	40°	39°	38°	37°	36°	35°	34°	′

NAT. COSINE.

′	48°	49°	50°	51°	52°	53°	54°	55°	′
0	1·1106125	1·1503684	1·1917536	1·2 48972	1·2799416	1·3270448	1·3763819	1·4281480	60
1	12624	10445	24579	56319	1·2807094	78483	72242	90326	59
2	19127	17210	31626	63672	14776	86524	80672	99178	58
3	25635	23979	38679	71030	22465	94571	89108	1·4308039	57
4	32146	30754	45736	78393	30160	1·3302624	97551	16900	56
5	38662	37532	52799	85762	37860	10684	1·3806001	25781	55
6	45182	44316	59866	93136	45566	18750	14458	34664	54
7	51706	51104	66938	1·2400515	53277	26822	22922	43554	53
8	58235	57896	74015	07900	60995	34900	31392	52451	52
9	64768	64693	81097	15290	68718	42984	39869	61356	51
10	71305	71495	88184	22685	76447	51075	48353	70268	50
11	77846	78301	95276	30086	84182	59172	56844	79187	49
12	84391	85112	1·2002373	37492	91922	67276	65342	88114	48
13	90941	91927	09475	44903	99669	75386	73847	97049	47
14	97495	98747	16581	52320	1·2907421	83502	82358	1·4405991	46
15	1·1204053	1·1605571	23693	59742	15179	91624	90876	14940	45
16	10616	12400	30810	67169	22913	99753	99401	23897	44
17	17183	19234	37932	74602	30713	1·3407888	1·3907934	32862	43
18	23754	26073	45058	82040	38488	16029	16473	41834	42
19	30329	32916	52190	89484	46270	24177	25019	50814	41
20	36909	39763	59327	96933	54057	32331	33571	59801	40
21	43493	46615	66468	1·2504388	61850	40492	42131	68796	39
22	50081	53472	73615	11848	69649	48658	50698	77798	38
23	56674	60334	80767	19313	77454	56832	59272	86808	37
24	63271	67200	87924	26784	85265	65011	67852	95825	36
25	69872	74071	95085	34260	93081	73198	76440	1·4504850	35
26	76478	80947	1·2102252	41742	1·3000904	81390	85034	13883	34
27	83088	87827	09424	49229	08733	89589	93636	22923	33
28	89702	94712	16601	56721	16567	97794	1·4002245	31971	32
29	96321	1·1701601	23783	64219	24407	1·3506006	10860	41027	31
30	1·1302944	08496	30970	71723	32254	14224	19483	50090	30
31	09571	15395	38162	79232	40106	22449	28113	59161	29
32	16203	22298	45359	86747	47964	30680	36749	68240	28
33	22839	29207	52562	94267	55828	38918	45393	77326	27
34	29479	36120	59769	1·2601792	63699	47162	54044	86420	26
35	36124	43038	66982	09323	71575	55413	62702	95522	25
36	42773	49960	74199	16860	79457	63670	71367	1·4604632	24
37	49427	56888	81422	24402	87345	71934	80039	13749	23
38	56085	63820	88650	31950	95239	80204	88718	22874	22
39	62747	70756	95883	39503	1·3103140	88481	97405	32007	21
40	69414	77698	·2203121	47062	11046	96764	1·4106098	41147	20
41	76086	84644	10364	54626	18958	1·3605054	14799	50290	19
42	82761	91595	17613	62196	26876	13350	23506	59452	18
43	89441	98551	24866	69772	34801	21653	32221	68616	17
44	96126	1·1805512	32125	77353	42731	29963	40943	77789	16
45	1·1402815	12477	39389	84940	50668	38279	49673	86967	15
46	09508	19447	46658	92532	58610	46602	58409	96155	14
47	16206	26422	53932	1·2700130	66559	54931	67153	1·4705350	13
48	22908	33402	61211	07733	74513	63267	75904	14553	12
49	29615	40387	68496	15342	82474	71610	84662	23764	11
50	36326	47376	75786	22957	90441	79959	93427	32983	10
51	43041	54370	83081	30578	98414	88315	1·4202200	42210	9
52	49762	61369	90381	38204	1·3206393	96678	10979	51445	8
53	56486	68373	97687	45835	14379	1·3705047	19766	60688	7
54	63215	75382	1·2304997	53473	22370	13423	28561	69938	6
55	69949	82395	12313	61116	30368	21806	37362	79197	5
56	76687	89414	19634	68765	38371	30195	46171	88463	4
57	83429	96437	26961	76419	46381	38591	54988	97738	3
58	90176	1·1903465	34292	84079	54397	46994	63811	1·4807021	2
59	96928	10498	41629	91745	62420	55403	72642	16311	1
60	1·1503684	17536	48972	99416	70448	63819	81480	25610	0
′	41°	40°	39°	38°	37°	36°	35°	34°	′

′	56°	57°	58°	59°	60°	61°	62°	63°	′
0	829 0376	838 6706	848 0481	857 1673	866 0254	874 6197	882 9476	891 0065	60
1	2002	8290	2022	3171	1708	7607	883 0841	1385	59
2	3628	9873	3562	4668	3161	9016	2206	2705	58
3	5252	839 1455	5102	6164	4614	875 0425	3569	4024	57
4	6877	3037	6641	7660	6066	1832	4933	5342	56
5	8500	4618	8179	9155	7517	3239	6295	6659	55
6	830 0123	6199	9717	858 0649	8967	4645	7656	7975	54
7	1745	7778	849 1254	2143	867 0417	6051	9017	9291	53
8	3366	9357	2790	3635	1866	7455	884 0377	892 0606	52
9	4987	840 0936	4325	5127	3314	8859	1736	1920	51
10	6607	2513	5860	6619	4762	876 0263	3095	3234	50
11	8226	4090	7394	8109	6209	1665	4453	4546	49
12	9845	5666	8927	9599	7655	3067	5810	5858	48
13	831 1463	7241	850 0459	859 1088	9100	4468	7166	7169	47
14	3080	8816	1991	2576	868 0544	5868	8522	8480	46
15	4696	841 0390	3522	4064	1988	7268	9876	9789	45
16	6312	1963	5053	5551	3431	8666	885 1230	893 1098	44
17	7927	3536	6582	7037	4874	877 0064	2584	2406	43
18	9541	5108	8111	8523	6315	1462	3936	3714	42
19	832 1155	6679	9639	860 0007	7756	2858	5288	5021	41
20	2768	8249	851 1167	1491	9196	4254	6639	6326	40
21	4380	9819	2693	2975	869 0636	5649	7989	7632	39
22	5991	842 1388	4219	4457	2074	7043	9339	8936	38
23	7602	2956	5745	5939	3512	8437	886 0688	894 0240	37
24	9212	4524	7269	7420	4949	9830	2036	1542	36
25	833 0822	6091	8793	8901	6386	878 1222	3383	2844	35
26	2430	7657	852 0316	861 0380	7821	2613	4730	4146	34
27	4038	9222	1839	1859	9256	4004	6075	5446	33
28	5646	843 0787	3360	3337	870 0691	5394	7420	6746	32
29	7252	2351	4881	4815	2124	6783	8765	8045	31
30	8858	3914	6402	6292	3557	8171	887 0108	9344	30
31	834 0463	5477	7921	7768	4989	9559	1451	895 0641	29
32	2068	7039	9440	9243	6420	879 0946	2793	1938	28
33	3672	8600	853 0958	862 0717	7851	2332	4134	3234	27
34	5275	844 0161	2475	2191	9281	3717	5475	4529	26
35	6877	1720	3992	3664	871 0710	5102	6815	5824	25
36	8479	3279	5508	5137	2138	6486	8154	7118	24
37	835 0080	4838	7023	6608	3566	7869	9492	8411	23
38	1680	6395	8538	8079	4993	9251	888 0830	9703	22
39	3279	7952	854 0051	9549	6419	880 0633	2166	896 0994	21
40	4878	9508	1564	863 1019	7844	2014	3503	2285	20
41	6476	845 1064	3077	2488	9269	3394	4838	3575	19
42	8074	2618	4588	3956	872 0693	4774	6172	4864	18
43	9670	4172	6099	5423	2116	6152	7506	6153	17
44	836 1266	5726	7609	6889	3538	7530	8839	7440	16
45	2862	7278	9119	8355	4960	8907	889 0171	8727	15
46	4456	8830	855 0627	9820	6381	881 0284	1503	897 0014	14
47	6050	846 0381	2135	864 1284	7801	1660	2834	1299	13
48	7643	1932	3643	2748	9221	3035	4164	2584	12
49	9236	3481	5149	4211	873 0640	4409	5493	3868	11
50	837 0827	5030	6655	5673	2058	5782	6822	5151	10
51	2418	6579	8160	7134	3475	7155	8149	6433	9
52	4009	8126	9664	8595	4891	8527	9476	7715	8
53	5598	9673	856 1168	865 0055	6307	9898	890 0803	8996	7
54	7187	847 1219	2671	1514	7722	882 1269	2128	898 0276	6
55	8775	2765	4173	2973	9137	2638	3453	1555	5
56	838 0363	4309	5674	4430	874 0550	4007	4777	2834	4
57	1950	5853	7175	5887	1963	5376	6100	4112	3
58	3536	7397	8675	7344	3375	6743	7423	5389	2
59	5121	8939	857 0174	8799	4786	8110	8744	6665	1
60	6706	848 0481	1673	866 0254	6197	9476	891 0065	7940	0
′	33°	32°	31°	30°	29°	28°	27°	26°	′

′	56°	57°	58°	59°	60°	61°	62°	63°	′
0	1·4825610	1·5398650	1·6003345	1·6642795	1·7320508	1·8040478	1·8807265	1·9626105	60
1	34916	1·5408460	13709	53766	32149	52860	20470	40227	59
2	44231	18280	24082	64748	43803	65256	33690	54364	58
3	53554	28108	34465	75741	55468	77664	46924	68518	57
4	62884	37946	44858	86744	67144	90086	60172	82688	56
5	72223	47792	55260	97758	78833	1·8102521	73436	96874	55
6	81570	57647	65672	1·6708782	90533	14969	86713	1·9711077	54
7	90925	67510	76094	19818	1·7402245	27430	1·8900006	25296	53
8	1·4900288	77383	86525	30864	13969	39904	13313	39531	52
9	09659	87264	96966	41921	25705	52391	26635	53782	51
10	19039	97155	1·6107417	52988	37453	64892	39971	68050	50
11	28426	1·5507054	17878	64067	49213	77405	53322	82334	49
12	37822	16963	28349	75156	60984	89932	66688	96635	48
13	47225	26880	38829	86256	72768	1·8202473	80068	1·9810952	47
14	56637	36806	49320	97367	84564	15026	93464	25286	46
15	66058	46741	59820	1·6808489	96371	27593	1·9006874	39636	45
16	75486	56685	70330	19621	1·7508191	40173	20299	54003	44
17	84923	66639	80850	30765	20023	52767	33738	68387	43
18	94367	76601	91380	41919	31866	65374	47193	82787	42
19	1·5003821	86572	1·6201920	53085	43722	77994	60663	97204	41
20	13282	96552	12469	64261	55590	90628	74147	1·9911637	40
21	22751	1·5606542	23029	75449	67470	1·8303275	87647	26087	39
22	32229	16540	3359[illegible]	86647	79362	15936	1·9101162	40554	38
23	41716	26548	44178	97856	91267	28610	14691	55038	37
24	51210	36564	54768	1·6909077	1·7603183	41297	28236	69539	36
25	60713	46590	65368	20308	15112	53999	41795	84056	35
26	70224	56625	75977	31550	27053	66713	55370	98590	34
27	79743	66669	86597	42804	39007	79442	68960	2·0013142	33
28	89271	76722	97227	54069	50972	92184	82565	27710	32
29	98807	86784	1·6307867	65344	62950	1·8404940	96186	42295	31
30	1·5108352	96856	18517	76631	74940	17709	1·9209821	56897	30
31	17905	1·5706936	29177	87929	86943	30492	23472	71516	29
32	27466	17026	39847	99238	98958	43289	37138	86153	28
33	37036	27120	50528	1·7010559	1·7710985	56099	50819	2·0100806	27
34	46614	37234	61218	21890	23024	68923	64516	15477	26
35	56201	47352	71919	33233	35076	81761	78228	30164	25
36	65796	57479	82630	44587	47141	94613	91956	44869	24
37	75400	67615	93351	55953	59218	1·8507479	1·9305699	59592	23
38	85012	77760	1·6404082	67329	71307	20358	19457	74331	22
39	94632	87915	14824	78717	83409	33252	33231	89068	21
40	1·5204261	98079	25576	90116	95524	46159	47020	2·0203862	20
41	13899	1·5808253	36338	1·7101527	1·7807651	59080	60825	18654	19
42	23545	18436	47111	12949	19790	72015	74645	33462	18
43	33200	28628	57893	24382	31943	84965	88481	48269	17
44	42863	38830	68687	35827	44107	97928	1·9402333	63133	16
45	52535	49041	79490	47283	56285	1·8610905	16200	77994	15
46	62215	59261	90304	58751	68475	23896	30083	92873	14
47	71904	69491	1·6501128	70230	80678	36902	43981	2·0 07769	13
48	81602	79731	11963	81720	92893	49921	57896	22683	12
49	91308	89979	22808	93222	1·7905121	62955	71826	37615	11
50	1·5301023	1·5900238	33663	1·7204736	17362	76003	85772	52565	10
51	10746	10505	44529	16261	29616	89065	99733	67532	9
52	20479	20783	55405	27797	41883	1·8702141	1·9513711	82517	8
53	30219	31070	66292	39346	54162	15231	27704	97519	7
54	39969	41366	77189	50905	66454	28336	41713	2·0412540	6
55	49727	51672	88097	62477	78759	41455	55739	27578	5
56	59494	61987	99016	74060	91077	54588	69780	42634	4
57	69270	72312	1·6609945	85654	1·8003408	67736	83837	57706	3
58	79054	82647	20884	97260	15751	80898	97910	72806	2
59	88849	92991	31834	1·7308878	28108	94074	1·9612000	87910	1
60	98650	1·6003345	42795	20508	40478	1·8807265	26105	2·0503038	0
′	33°	32°	31°	30°	29°	28°	27°	26°	′

NAT. COTAN.

′	64°	65°	66°	67°	68°	69°	70°	71°	′
0	898 7940	906 3078	913 5455	920 5049	927 1839	933 5804	939 6926	945 5186	60
1	9215	4307	6637	6185	2928	6846	7921	6132	59
2	899 0489	5535	7819	7320	4016	7888	8914	7078	58
3	1763	6762	9001	8455	5104	8928	9907	8023	57
4	3035	7989	914 0181	9589	6191	9968	940 0899	8968	56
5	4307	9215	1361	921 0722	7277	934 1007	1891	9911	55
6	5578	907 0440	2540	1854	8363	2045	2881	946 0854	54
7	6848	1665	3718	2986	9447	3082	3871	1795	53
8	8117	2888	4895	4116	928 0531	4119	4860	2736	52
9	9386	4111	6072	5246	1614	5154	5848	3677	51
10	900 0654	5333	7247	6375	2696	6189	6835	4616	50
11	1921	6554	8422	7504	3778	7223	7822	5555	49
12	3188	7775	9507	8632	4858	8257	8808	6493	48
13	4453	8995	915 0770	9758	5938	9289	9793	7430	47
14	5718	908 0214	1943	922 0884	7017	935 0321	941 0777	8366	46
15	6982	1432	3115	2010	8096	1352	1760	9301	45
16	8246	2649	4286	3134	9173	2382	2743	947 0236	44
17	9508	3866	5456	4258	929 0250	3412	3724	1170	43
18	901 0770	5082	6626	5381	1326	4440	4705	2105	42
19	2031	6297	7795	6503	2401	5468	5686	3035	41
20	3292	7511	8963	7624	3475	6495	6665	3966	40
21	4551	8725	916 0130	8745	4549	7521	7644	4897	39
22	5810	9938	1297	9865	5622	8547	8621	5827	38
23	7068	909 1150	2462	923 0984	6694	9571	9598	6756	37
24	8325	2361	3627	2102	7765	936 0595	942 0575	7684	36
25	9582	3572	4791	3220	8835	1618	1550	8612	35
26	902 0838	4781	5955	4336	9905	2641	2525	9538	34
27	2092	5990	7118	5452	930 0974	3662	3498	948 0464	33
28	3347	7199	8279	6567	2042	4683	4471	1389	32
29	4600	8406	9440	7682	3109	5703	5444	2313	31
30	5853	9613	917 0601	8795	4176	6722	6415	3237	30
31	7105	910 0819	1760	9908	5241	7740	7386	4159	29
32	8356	2024	2919	924 1020	6306	8758	8355	5081	28
33	9606	3228	4077	2131	7370	9774	9324	6002	27
34	903 0856	4432	5234	3242	8434	937 0790	943 0293	6922	26
35	2105	5635	6391	4351	9496	1806	1260	7842	25
36	3353	6837	7546	5460	931 0558	2820	2227	8760	24
37	4600	8038	8701	6568	1619	3833	3192	9678	23
38	5847	9238	9855	7676	2679	4846	4157	949 0595	22
39	7093	911 0438	918 1009	8782	3739	5858	5122	1511	21
40	8338	1637	2161	9888	4797	6869	6085	2426	20
41	9582	2835	3313	925 0993	5855	7880	7048	3341	19
42	904 0825	4033	4464	2097	6912	8889	8010	4255	18
43	2068	5229	5614	3201	7969	9898	8971	5168	17
44	3310	6425	6763	4303	9024	938 0906	9931	6080	16
45	4551	7620	7912	5405	932 0079	1913	944 0890	6991	15
46	5792	8815	9060	6506	1133	2920	1849	7902	14
47	7032	912 0008	919 0207	7606	2186	3925	2807	8812	13
48	8271	1201	1353	8706	3238	4930	3764	9721	12
49	9509	2393	2499	9805	4290	5934	4720	950 0629	11
50	905 0746	3584	3644	926 0902	5340	6938	5675	1536	10
51	1983	4775	4788	2000	6390	7940	6630	2443	9
52	3219	5965	5931	3096	7439	8942	7584	3348	8
53	4454	7154	7073	4192	8488	9943	8537	4253	7
54	5688	8342	8215	5286	9535	939 0943	9489	5157	6
55	6922	9529	9356	6380	933 0582	1942	945 0441	6061	5
56	8154	913 0716	920 0496	7474	1628	2940	1391	6963	4
57	9386	1902	1635	8566	2673	3938	2341	7865	3
58	906 0618	3087	2774	9658	3718	4935	3290	8766	2
59	1848	4271	3912	927 0748	4761	5931	4238	9666	1
60	3078	5455	5049	1839	5804	6926	5186	951 0565	0
′	25°	24°	23°	22°	21°	20°	19°	18°	′

′	64°	65°	66°	67°	68°	69°	70°	71°	′
0	2·0503038	2·1445069	2·2460368	2·3558524	2·4750869	2·6050891	2·7474774	2·9042109	60
1	18185	61366	77962	77590	71612	73558	99661	69576	59
2	33349	77683	95580	96683	92386	96259	2·7524588	97089	58
3	48531	94021	2·2513221	2·3615801	2·4813190	2·6118995	49554	2·9124649	57
4	63737	2·1510378	30885	34946	34023	41766	74561	52256	56
5	78950	26757	48572	54118	54887	64571	99608	79909	55
6	94187	43156	66283	73316	75781	87411	2·7624695	2·9207610	54
7	2·0609442	59575	84016	92540	96706	2·6210286	49822	35358	53
8	24716	76015	2·2601773	2·3711791	2·4917660	33196	74990	63152	52
9	40008	92476	19554	31068	38645	56141	2·7700199	90995	51
10	55318	2·1608958	37357	50372	59661	79121	25448	2·9318885	50
11	70646	25460	55184	69703	80707	2·6302136	50738	46822	49
12	85994	41983	73035	89060	2·5001784	25186	76069	74807	48
13	2·0701359	58527	90909	2·3808444	22891	48271	2·7801440	2·9402840	47
14	16743	75091	2·2708807	27855	44029	71392	26853	30921	46
15	32146	91677	26729	47293	65198	94549	52307	59050	45
16	47567	2·1708283	44674	66758	86398	2·6417741	77802	87227	44
17	63007	24911	62643	86250	2·5107629	40969	2·7903339	2·9515453	43
18	78465	41559	80636	2·3905769	28890	64232	28917	43727	42
19	93942	58229	98653	25316	50183	87531	54537	72050	41
20	2·0809438	74920	2·2816693	44889	71507	2·6510867	80198	2·9600422	40
21	24953	91631	34758	64490	92863	34238	2·8005901	28842	39
22	40487	2·1808364	52846	84118	2·5214249	57645	31646	57312	38
23	56039	25119	70959	2·4003774	35667	81089	57433	85831	37
24	71610	41894	89096	23457	57117	2·6604569	83263	2·9714399	36
25	87200	58691	2·2907257	43168	78598	28085	2·8109164	43016	35
26	2·0902809	75510	25442	62906	2·5300111	51638	35048	71683	34
27	18437	92349	43651	82672	21655	75227	61004	2·9800400	33
28	34085	2·1909210	61885	2·4102465	43231	98853	87003	29167	32
29	49751	26093	80143	22286	64839	2·6722516	2·8213045	57983	31
30	65436	42997	98425	42136	86479	46215	39129	86850	30
31	81140	59923	2·3016732	62013	2·5408151	69951	65256	2·9915766	29
32	96864	76871	35064	81918	29855	93725	91426	44734	28
33	2·1012607	93840	53420	2·4201851	51591	2·6817535	2·8317639	73751	27
34	28369	2·2010831	71801	21812	73359	41383	43890	3·0002820	26
35	44150	27843	90206	41801	95160	65267	70190	31939	25
36	59951	44878	2·3108637	61819	2·5516992	89190	96539	61109	24
37	75771	61934	27092	81864	38858	2·6913149	2·8422926	90330	23
38	91611	79012	45571	2·4301938	60756	37147	49356	3·0119603	22
39	2·1107470	96112	64076	22041	82686	61181	75831	48926	21
40	23348	2·2113234	82606	42172	2·5604649	85254	2·8502349	78301	20
41	39246	30379	2·3201160	62331	26645	2·7009364	28911	3·0287728	19
42	55164	47545	19740	82519	48674	33513	55517	37207	18
43	71101	64733	38345	2·4402736	70735	57699	82168	66737	17
44	87057	81944	56975	22982	92830	81923	2·8608863	96320	16
45	2·1203034	99177	75630	43256	2·5714957	2·7106186	35602	3·0325954	15
46	19030	2·2216432	94311	63559	37118	30487	62386	55641	14
47	35046	33709	2·3313017	83891	59312	54826	89215	85381	13
48	51082	51009	31748	2·4504252	81539	79204	2·8716088	3·0415173	12
49	67137	68331	50505	24642	2·5803800	2·7203620	43007	45018	11
50	83213	85676	69287	45061	26094	28076	69970	74915	10
51	99308	2·2303043	88095	65510	48421	52569	96979	3·0504866	9
52	2·1315423	20433	2·3406928	85987	70782	77102	2·8824033	34870	8
53	31559	37845	25787	2·4606494	93177	2·7301674	51132	64928	7
54	47714	55280	44672	27030	2·5915606	26284	78277	95038	6
55	63890	72738	63582	47596	38068	50934	2·8905467	3·0625203	5
56	80085	90218	82519	68191	60564	75623	32704	55421	4
57	96301	2·2407721	2·3501481	88816	83095	2·7400352	59986	85694	3
58	2·1412537	25247	20469	2·4709470	2·6005659	25120	87314	3·0716020	2
59	28793	42796	39483	30155	28258	49927	2·9014688	46400	1
60	45069	60368	58524	50869	50891	74774	42109	76835	0
′	25°	24°	23°	22°	21°	20°	19°	18°	′

′	72°	73°	74°	75°	76°	77°	78°	79°	′
0	951 0565	956 3048	961 2617	965 9258	970 2957	974 3701	978 1476	981 6272	60
1	1464	3898	3418	966 0011	3661	4355	2080	6826	59
2	2361	4747	4219	0762	4363	5008	2684	7380	58
3	3258	5595	5019	1513	5065	5660	3287	7933	57
4	4154	6443	5818	2263	5766	6311	3889	8485	56
5	5050	7290	6616	3012	6466	6962	4490	9037	55
6	5944	8136	7413	3761	7165	7612	5090	9587	54
7	6838	8981	8210	4508	7863	8261	5689	982 0137	53
8	7731	9825	9005	5255	8561	8909	6288	0686	52
9	8623	957 0669	9800	6001	9258	9556	6886	1234	51
10	9514	1512	962 0594	6746	9953	975 0203	7483	1781	50
11	952 0404	2354	1387	7490	971 0649	0849	8079	2327	49
12	1294	3195	2180	8234	1343	1494	8674	2873	48
13	2183	4035	2972	8977	2036	2138	9268	3417	47
14	3071	4875	3762	9718	2729	2781	9862	3961	46
15	3958	5714	4552	967 0459	3421	3423	979 0455	4504	45
16	4844	6552	5342	1200	4112	4065	1047	5046	44
17	5730	7389	6130	1939	4802	4706	1638	5587	43
18	6615	8225	6917	2678	5491	5345	2228	6128	42
19	7499	9060	7704	3415	6180	5985	2818	6668	41
20	8382	9895	8490	4152	6867	6623	3406	7206	40
21	9264	958 0729	9275	4888	7554	7260	3994	7744	39
22	953 0146	1562	963 0060	5624	8240	7897	4581	8282	38
23	1027	2394	0843	6358	8926	8533	5167	8818	37
24	1907	3226	1626	7092	9610	9168	5752	9353	36
25	2786	4056	2408	7825	972 0294	9802	6337	9888	35
26	3664	4886	3189	8557	0976	976 0435	6921	983 0422	34
27	4542	5715	3969	9288	1658	1068	7504	0955	33
28	5418	6543	4748	968 0018	2339	1699	8086	1487	32
29	6294	7371	5527	0748	3020	2330	8668	2019	31
30	7170	8197	6305	1476	3699	2960	9247	2549	30
31	8044	9023	7081	2204	4378	3589	9827	3079	29
32	8917	9848	7858	2931	5056	4218	980 0405	3608	28
33	9790	959 0672	8633	3658	5733	4845	0983	4136	27
34	954 0662	1496	9407	4383	6409	5472	1560	4663	26
35	1533	2318	964 0181	5108	7084	6098	2136	5189	25
36	2403	3140	0954	5832	7759	6723	2712	5715	24
37	3273	3961	1726	6555	8432	7347	3286	6239	23
38	4141	4781	2497	7277	9105	7970	3860	6763	22
39	5009	5600	3268	7998	9777	8593	4433	7286	21
40	5876	6418	4037	8719	973 0449	9215	5005	7808	20
41	6743	7236	4806	9438	1119	9836	5576	8330	19
42	7608	8053	5574	969 0157	1789	977 0456	6147	8850	18
43	8473	8869	6341	0875	2458	1075	6716	9370	17
44	9336	9684	7108	1593	3125	1693	7285	9889	16
45	955 0199	960 0499	7873	2309	3793	2311	7853	984 0407	15
46	1062	1312	8638	3025	4459	2928	8420	0924	14
47	1923	2125	9402	3740	5124	3544	8986	1441	13
48	2784	2937	965 0165	4453	5789	4159	9552	1956	12
49	3643	3748	0927	5167	6453	4773	981 0116	2471	11
50	4502	4558	1689	5879	7116	5387	0680	2985	10
51	5361	5368	2449	6591	7778	5999	1243	3498	9
52	6218	6177	3209	7301	8439	6611	1805	4010	8
53	7074	6984	3968	8011	9100	7222	2366	4521	7
54	7930	7792	4726	8720	9760	7832	2927	5032	6
55	8785	8598	5484	9428	974 0419	8442	3486	5542	5
56	9639	9403	6240	970 0136	1077	9050	4045	6050	4
57	956 0492	961 0208	6996	0842	1734	9658	4603	6558	3
58	1345	1012	7751	1548	2390	978 0265	5160	7066	2
59	2197	1815	8505	2253	3046	0871	5716	7572	1
60	3048	2617	9258	2957	3701	1476	6272	8078	0
′	17°	16°	15°	14°	13°	12°	11°	10°	′

NAT. COSINE.

′	72°	73°	74°	75°	76°	77°	78°	′
0	3·0776835	3·2708526	3·4874144	3·7320508	4·0107809	4·3314759	4·7046301	60
1	3·0807325	42588	3·4912470	63980	57570	72316	4·7113686	59
2	37869	76715	50874	3·7407546	4·0207446	4·3430018	81256	58
3	68468	3·2810907	89356	51207	57440	87866	4·7249012	57
4	99122	45164	3·5027916	94963	4·0307550	4·3545861	4·7316954	56
5	3·0929831	79487	66555	3·7538815	57779	4·3604003	85083	55
6	60596	3·2913876	3·5105273	82763	4·0408125	62293	4·7453401	54
7	91416	48330	44070	3·7626807	58590	4·3720731	4·7521907	53
8	3·1022291	82851	82946	70947	4·0509174	79317	90603	52
9	53223	3·3017438	3·5221902	3·7715185	59877	4·3838054	4·7659490	51
10	84210	52091	60938	59519	4·0610700	96940	4·7728568	50
11	3·1115254	86811	3·5300054	3·7803951	61643	4·3955977	97837	49
12	46353	3·3121598	39251	48481	4·0712707	4·4015164	4·7867300	48
13	77509	56452	78528	93109	63892	74504	4·7936957	47
14	3·1208722	91373	3·5417836	3·7937835	4·0815199	4·4133996	4·8006808	46
15	39991	3·3226362	57325	82661	66627	93641	76854	45
16	71317	61419	96846	3·8027585	4·0918178	4·4253439	4·8147096	44
17	3·1302701	96543	3·5526449	72609	69852	4·4313392	4·8217536	43
18	34141	3·3331736	76133	3·8117733	4·1021649	73500	88174	42
19	65639	66997	3·5615900	62957	73569	4·4433762	4·8359010	41
20	97194	3·3402326	55749	3·8208281	4·1125614	94181	4·8430045	40
21	3·1428807	37724	95681	53707	77784	4·4554756	4·8501262	39
22	60478	73191	3·5735696	99233	4·1230079	4·4615489	72719	38
23	92207	3·3508728	75794	3·8344861	82499	76379	4·8644359	37
24	3·1523994	44333	3·5815975	90591	4·1335046	4·4737428	4·8716201	36
25	55840	80008	56241	3·8436424	87719	98636	88248	35
26	87744	3·3615753	96590	82358	4·1440519	4·4860004	4·8860499	34
27	3·1619706	51568	3·5937024	3·8528396	93446	4·4921532	4·8932956	33
28	51728	87453	77543	74537	4·1546501	83221	4·9005620	32
29	83808	3·3723408	3·6018146	3·8620782	99685	4·5045072	78491	31
30	3·1715948	59434	58835	67131	4·1652998	4·5107085	4·9151570	30
31	48147	95531	99609	3·8713584	4·1706440	69261	4·9224859	29
32	80406	3·3831699	3·6140469	60142	60011	4·5231601	98358	28
33	3·1812724	67938	81415	3·8806805	4·1813713	94105	4·9372068	27
34	45102	3·3904249	3·6222447	53574	67546	4·5356773	4·9445990	26
35	77540	40631	63566	3·8900448	4·1921510	4·5419608	4·9520125	25
36	3·1910039	77085	3·6304771	47429	75606	82608	94474	24
37	42598	3·4013612	46064	94516	4·2029835	4·5545776	4·9669037	23
38	75217	50210	87444	3·9041710	84196	4·5609111	4·9743817	22
39	3·2007897	86882	3·6428911	89011	4·2138690	72615	4·9818813	21
40	40638	3·4123626	70467	3·9136420	93318	4·5736287	94027	20
41	73440	60443	3·6512111	83937	4·2248080	4·5800129	4·9969459	19
42	3·2106304	97333	53844	3·9231563	4·2302977	64141	5·0045111	18
43	39228	3·4234297	95665	79297	58009	4·5928325	5·0120984	17
44	72215	71334	3·6637575	3·9327141	4·2413177	92680	97078	16
45	3·2205263	3·4308446	79575	75094	68482	4·6057207	5·0273395	15
46	38373	45631	3·6721665	3·9423157	4·2523923	4·6121908	5·0349935	14
47	71546	82891	63845	71331	79501	86783	5·0426700	13
48	3·2304780	3·4420226	3·6806115	3·9519615	4·2635218	4·6251832	5·0503690	12
49	38078	57635	48475	68011	91072	4·6317056	80907	11
50	71438	95120	90927	3·9616518	4·2747066	82457	5·0658352	10
51	3·2404860	3·4532679	3·6933469	65137	4·2803199	4·6448031	5·0736025	9
52	38346	70315	76104	3·9713868	59472	4·6513788	5·0813928	8
53	71895	3·4608026	3·7018830	62712	4·2915885	79721	92061	7
54	3·2505508	45813	61648	3·9811669	72440	4·6645832	5·0970426	6
55	39184	83676	3·7104558	60739	4·3029136	4·6712124	5·1049024	5
56	72924	3·4721616	47561	3·9909924	85974	78595	5·1127855	4
57	3·2606728	59632	90658	59223	4·3142955	4·6845248	5·1206921	3
58	40596	97726	3·7233847	4·0008636	4·3200079	4·6912083	86224	2
59	74529	3·4835896	77131	58165	57347	79100	5·1365763	1
60	3·2708526	74144	3·7320508	4·0107809	4·3314759	4·7046301	5·1445540	0
′	17°	16°	15°	14°	13°	12°	11°	′

NAT. COTAN.

′	80°	81°	82°	83°	84°	85°	86°	87°	′
0	9848 078	9876 883	9902 681	9925 462	9945 219	9961 947	9975 641	9986 295	60
1	582	9877 338	9903 085	816	523	9962 200	843	447	59
2	9849 086	792	489	9926 169	825	452	9976 045	598	58
3	589	9878 245	891	521	9946 127	704	245	748	57
4	9850 091	697	9904 293	873	428	954	445	898	56
5	593	9879 148	694	9927 224	729	9963 204	645	9987 046	55
6	9851 093	599	9905 095	573	9947 028	453	843	194	54
7	593	9880 048	494	922	327	701	9977 040	340	53
8	9852 092	497	893	9928 271	625	948	237	486	52
9	590	945	9906 290	618	921	9964 195	433	631	51
10	9853 087	9881 392	687	965	9948 217	440	627	775	50
11	583	838	9907 083	9929 310	513	685	821	919	49
12	9854 079	9882 284	478	655	807	929	9978 015	9988 061	48
13	574	728	873	999	9949 101	9965 172	207	203	47
14	9855 068	9883 172	9908 266	9930 342	393	414	399	344	46
15	561	615	659	685	685	655	589	484	45
16	9856 053	9884 057	9909 051	9931 026	976	895	779	623	44
17	544	498	442	367	9950 266	9966 135	968	761	43
18	9857 035	939	832	706	556	374	9979 156	899	42
19	524	9885 378	9910 221	9932 045	844	612	343	9989 035	41
20	9858 013	817	610	384	9951 132	849	530	171	40
21	501	9886 255	997	721	419	9967 085	716	306	39
22	988	692	9911 384	9933 057	705	321	900	440	38
23	9859 475	9887 128	770	393	990	555	9980 084	573	37
24	960	564	9912 155	728	9952 274	789	267	706	36
25	9860 445	998	540	9934 062	557	9968 022	450	837	35
26	929	9888 432	923	395	840	254	631	968	34
27	9861 412	865	9913 306	727	9953 122	485	811	9990 098	33
28	894	9889 297	688	9935 058	403	715	991	227	32
29	9862 375	728	9914 069	389	683	945	9981 170	355	31
30	856	9890 159	449	719	962	9969 173	348	482	30
31	9863 336	588	828	9936 047	9954 240	401	525	609	29
32	815	9891 017	9915 206	375	518	628	701	734	28
33	9864 293	445	584	703	795	854	877	859	27
34	770	872	961	9937 029	9955 070	9970 080	9982 052	983	26
35	9865 246	9892 298	9916 337	355	345	304	225	9991 106	25
36	722	723	712	679	620	528	398	228	24
37	9866 196	9893 148	9917 086	9938 003	893	750	570	350	23
38	670	572	459	326	9956 165	972	742	470	22
39	9867 143	994	832	648	437	9971 193	912	590	21
40	615	9894 416	9918 204	969	708	413	9983 082	709	20
41	9868 087	838	574	9939 290	978	633	250	827	19
42	557	9895 258	944	610	9957 247	851	418	944	18
43	9869 027	677	9919 314	928	515	9972 069	585	9992 060	17
44	496	9896 096	682	9940 246	783	286	751	176	16
45	964	514	9920 049	563	9958 049	502	917	290	15
46	9870 431	931	416	880	315	717	9984 081	404	14
47	897	9897 347	782	9941 195	580	931	245	517	13
48	9871 363	762	9921 147	510	844	9973 145	408	629	12
49	827	9898 177	511	823	9959 107	357	570	740	11
50	9872 291	590	874	9942 136	370	569	731	851	10
51	754	9899 003	9922 237	448	631	780	891	960	9
52	9873 216	415	599	760	892	990	9985 050	9993 069	8
53	678	826	959	9943 070	9960 152	9974 199	209	177	7
54	9874 138	9900 237	9923 319	379	411	408	367	284	6
55	598	646	679	688	669	615	524	390	5
56	9875 057	9901 055	9924 037	996	926	822	680	495	4
57	514	462	394	9944 303	9961 183	9975 028	835	600	3
58	972	869	751	609	438	233	989	704	2
59	9876 428	9902 275	9925 107	914	693	437	9986 143	806	1
60	883	681	462	9945 219	947	641	295	908	0
′	9°	8°	7°	6°	5°	4°	3°	2°	′

NAT. COSINE.

′	79°	80°	81°	82°	83°	84°	85°	′
0	5·1445540	5·6712818	6·3137515	7·1153697	8·1443464	9·5143645	11·430052	60
1	525557	809446	256601	304190	639786	410613	468474	59
2	605813	906394	376126	455308	837041	679068	507154	58
3	686311	5·7003663	496092	607056	8·2035239	949022	546093	57
4	767051	101256	616502	759437	234384	9·6220486	585294	56
5	848035	199173	737359	912456	434485	493475	624761	55
6	929264	297416	858665	7·2066116	635547	768000	664495	54
7	5·2010738	395988	980422	220422	837579	9·7044075	704500	53
8	092459	414889	6·4102633	375378	8·3040586	321713	744779	52
9	174428	594122	225301	530987	244577	600927	785333	51
10	256647	693688	348428	687255	449558	881732	826167	50
11	339116	793588	472017	844184	655536	9·8164140	867282	49
12	421836	893825	596070	7·3001780	862519	448166	908682	48
13	504809	994400	720591	160047	8·4070515	733823	950370	47
14	588035	5·8095315	845581	318989	279531	9·9021125	992349	46
15	671517	196572	971043	478610	489573	310088	12·034622	45
16	755255	298172	6·5096981	638916	700651	600724	077192	44
17	839251	400117	223396	799909	912772	893050	120062	43
18	923505	502410	350293	961595	8·5125943	10·018708	163236	42
19	5·3008018	605051	477672	7·4123978	340172	048283	206716	41
20	092793	708042	605538	287064	555468	078031	250505	40
21	177830	811386	733892	450855	771838	107954	294609	39
22	263131	915084	862739	615357	989290	138054	339028	38
23	348696	5·9019138	992080	780576	8·6207833	168332	383768	37
24	434527	123550	6·6121919	946514	427475	198789	428831	36
25	520626	228322	252258	7·5113178	648223	229428	474221	35
26	606993	333455	383100	280571	870088	260249	519942	34
27	693630	438952	514449	448699	8·7093077	291255	565997	33
28	780538	544815	646307	617567	317198	322447	612390	32
29	867718	651045	778677	787179	542461	353827	659125	31
30	955172	757644	911562	957541	768874	385397	706205	30
31	5·4042901	864614	6·7044966	7·6128657	996446	417158	753634	29
32	130906	971957	178891	300533	8·8225186	449112	801417	28
33	219188	6·0079676	313341	473174	455103	481261	849557	27
34	307750	187772	448318	646584	686206	513607	898058	26
35	396592	296247	583826	820769	918505	546151	946924	25
36	485715	405103	719867	995735	8·9152009	578895	996160	24
37	575121	514343	856446	7·7171486	386726	611841	13·045769	23
38	664812	623967	993565	348028	622668	644992	095757	22
39	754788	733979	6·8131227	525366	859843	678348	146127	21
40	845052	844381	269437	703506	9·0098261	711913	196883	20
41	935604	955174	408196	882453	337933	745687	248031	19
42	5·5026446	6·1066360	547508	7·8062212	578867	779673	299574	18
43	117579	177943	687378	242790	821074	813872	351518	17
44	209005	289923	827807	424191	9·1064564	848288	403867	16
45	300724	402303	968799	606423	309348	882921	456625	15
46	392740	515085	6·9110359	789489	555436	917775	599799	14
47	485052	628272	252489	973396	802838	952850	563391	13
48	577663	741865	395192	7·9158151	9·2051564	988150	617409	12
49	670574	855867	538473	343758	301627	11·023676	671856	11
50	763786	970279	682335	530224	553035	059431	726738	10
51	857302	6·2085106	826781	717555	805802	095416	782060	9
52	951121	200347	971806	905756	9·3059936	131635	837827	8
53	5·6045247	316007	7·0117441	8·0094835	315450	168089	894045	7
54	139680	432086	263662	284796	572355	204780	950719	6
55	234421	548588	410482	475647	830663	241712	14·007856	5
56	329474	665515	557905	667394	9·4090384	278885	065459	4
57	424838	782868	705934	860042	351531	316304	123536	3
58	520516	900651	854573	8·1053599	614116	353970	182092	2
59	616509	6·3018866	7·1003826	248071	878149	391885	241134	1
60	712818	137515	153697	443464	9·5143645	430052	300666	0
′	10°	9°	8°	7°	6°	5°	4°	′

NAT. SINE.

′	88°	89°	′
0	9993 908	9998 477	60
1	9994 009	527	59
2	110	577	58
3	209	625	57
4	308	673	56
5	405	720	55
6	502	766	54
7	598	812	53
8	693	856	52
9	788	900	51
10	881	942	50
11	974	984	49
12	9995 066	9999 025	48
13	157	065	47
14	247	105	46
15	336	143	45
16	424	181	44
17	512	218	43
18	599	254	42
19	684	289	41
20	770	323	40
21	854	357	39
22	937	389	38
23	9996 020	421	37
24	101	452	36
25	182	482	35
26	262	511	34
27	341	539	33
28	419	567	32
29	497	593	31
30	573	619	30
31	649	644	29
32	724	668	28
33	798	692	27
34	871	714	26
35	943	736	25
36	9997 015	756	24
37	086	776	23
38	156	795	22
39	224	813	21
40	292	831	20
41	360	847	19
42	426	863	18
43	492	878	17
44	556	892	16
45	620	905	15
46	683	917	14
47	745	928	13
48	807	939	12
49	867	949	11
50	927	958	10
51	986	966	9
52	9998 044	973	8
53	101	979	7
54	157	985	6
55	213	989	5
56	267	993	4
57	321	996	3
58	374	998	2
59	426	1·0000 000	1
60	477	000	0
′	1°	0°	′

NAT. COSINE.

NAT. TAN.

′	86°	87°	88°	89°	′
0	14·300666	19·081137	28·636253	57·289962	60
1	360696	187930	877089	58·261174	59
2	421230	295922	29·122005	59·265872	58
3	482273	405133	371106	60·305820	57
4	543833	515584	624499	61·382905	56
5	605916	627296	882299	62·499154	55
6	668529	740291	30·144619	63·656741	54
7	731679	854591	411580	64·858008	53
8	795372	970219	683307	66·105473	52
9	859616	20·087199	959928	67·401854	51
10	924417	205553	31·241577	68·750087	50
11	989784	325308	528392	70·153346	49
12	15·055723	446486	820516	71·615070	48
13	122242	569115	32·118099	73·138991	47
14	189349	693220	421295	74·729165	46
15	257052	818828	730264	76·390009	45
16	325358	945966	33·045173	78·126342	44
17	394276	21·074664	366194	79·943430	43
18	463814	204949	693509	81·847041	42
19	533981	336851	34·027303	83·843507	41
20	604784	470401	367771	85·939791	40
21	676233	605630	715115	88·143572	39
22	748337	742569	35·069546	90·463336	38
23	821105	881251	431282	92·908487	37
24	894545	22·021710	800553	95·489475	36
25	968667	163980	36·177596	98·217943	35
26	16·043482	308097	562659	101·10690	34
27	118998	454096	956001	104·17094	33
28	195225	602015	37·357892	107·42648	32
29	272174	751892	768613	110·89205	31
30	349855	903766	38·188459	114·58865	30
31	428279	23·057677	617738	118·54018	29
32	507456	213666	39·056771	122·77396	28
33	587396	371777	505895	127·32134	27
34	668112	532052	965460	132·21851	26
35	749614	694537	40·435837	137·50745	25
36	831915	859277	917412	143·23712	24
37	915025	24·026320	41·410588	149·46502	23
38	998957	195714	915790	156·25908	22
39	17·083724	367509	42·433464	163·70019	21
40	169337	541758	964077	171·88540	20
41	255809	718512	43·508122	180·93220	19
42	343155	897826	44·066113	190·98419	18
43	431385	25·079757	638596	202·21875	17
44	520516	264361	45·226141	214·85762	16
45	610559	451700	829351	229·18166	15
46	701529	641832	46·448862	245·55198	14
47	793442	834823	47·085343	264·44080	13
48	886310	26·030736	739501	286·47773	12
49	980150	229638	48·412084	312·52137	11
50	18·074977	431600	49·103881	343·77371	10
51	170807	636690	815726	381·97099	9
52	267654	844984	50·548506	429·71757	8
53	365537	27·056557	51·303157	491·10600	7
54	464471	271486	52·080673	572·95721	6
55	564473	489853	882109	687·54887	5
56	665562	711740	53·708587	859·43630	4
57	767754	937233	54·561300	1145·9153	3
58	871068	28·166422	55·441517	1718·8732	2
59	975523	399397	56·350590	3437·7467	1
60	19·081137	636253	57·289962	Infinite.	0
′	3°	2°	1°	0°	′

NAT. COTAN.

TABLE OF RADII, &c.—Chord 100 Feet.

Angle of Deflex'n.	Radius in feet.	Def. dist. in feet.	Angle of Deflex'n.	Radius in feet.	Def. dist. in feet.	Angle of Deflex'n.	Radius in feet.	Def. dist in feet.
° ′			° ′			° ′		
5	68760·0	·145	3 25	1677·0	5·962	12 15	468·7	21.360
10	34380·0	·291	30	1637·0	6·108	30	459·3	21·790
15	22920·0	·436	35	1599·0	6·253	45	450·3	22·210
20	17190·0	·581	40	1563·0	6·398	13°	441·7	22·640
25	13752·0	·727	45	1528·0	6·544	15	433·4	23·070
30	11460·0	·872	50	1495·0	6·689	30	425·5	23·510
35	9823·0	1·017	55	1463·0	6·835	45	417·7	23·940
40	8595·0	1·163	4°	1433.0	6·980	14°	410·3	24·370
45	7640·0	1·308	15	1348·0	7·416	15	403·1	24·810
50	6876·0	1·453	30	1274·0	7·853	30	396·2	25·240
55	6251·0	1·600	45	1207·0	8·289	45	389·6	25·670
1°	5730·0	1·745	5°	1146·0	8·722	15°	383·1	26·110
5	5289·0	1·890	15	1092·0	9·159	15	376·9	26·520
10	4912·0	2·036	30	1042·0	9·595	30	370·8	26·940
15	4584·0	2·181	45	996·8	10·030	45	365·0	27·370
20	4298·0	2·327	6°	955·4	10·470	16°	359·3	27·830
25	4045·0	2·472	15	917·0	10·900	30	348·4	28·700
30	3820·0	2·618	30	882·0	11·340	17°	338·3	29·560
35	3619·0	2·763	45	849·3	11·780	30	328·7	30·430
40	3438·0	2·908	7°	819·0	12·210	18°	319·6	31·290
45	3274·0	3·054	15	790·8	12·640	30	311·0	32·500
50	3125·0	3·199	30	764·5	13·080	19°	302·9	33·010
55	2990·0	3·345	45	739·9	13·510	30	295·3	33·870
2°	2865·0	3·490	8°	716·8	13·950	20°	287·9	34·730
5	2750·0	3·635	15	695·1	14·380	21	274.4	36·440
10	2644·0	3·781	30	674·6	14·810	22	262·0	38·150
15	2547·0	3·926	45	655·5	15·250	23	250·8	39·870
20	2456·0	4·072	9°	637·3	15·680	24	240·5	41·580
25	2371·0	4·217	15	620·2	16·120	25	231·0	43·280
30	2292·0	4·363	30	603·8	16·550	26	222·3	44·980
35	2218·0	4·508	45	588·4	16·990	27	214·2	46·680
40	2149·0	4·653	10°	573·7	17·430	28	206·7	48·380
45	2084·0	4·799	15	559·7	17·870	29	199·7	50·070
50	2023·0	4·944	30	546.4	18·300	30	193·2	51·760
55	1965·0	5·090	45	533·8	18·730	31	187·1	53·450
3°	1910·0	5·235	11°	521·7	19·170	32	181·4	55·130
5	1859·0	5·380	15	510·1	19·610	33	176·0	56·800
10	1810·0	5·526	30	499·1	20·050	34	171·0	58·470
15	1763·0	5·671	45	488·5	20·500	35	166·3	60·140
20	1719·0	5·817	12°	478·3	20·940	36	161·8	61·800

TABLE OF LONG CHORDS.

Radius in feet.	Angle of Deflection.	Length of Chord in feet required to subtend 1 Station.	2 Stations.	3 Stations.	4 Stations.
5730·0	1°	100	200·0	300·0	400·0
4584·0	¼	100	200·0	300·0	399·9
3820·0	½	100	200·0	300·0	399·9
3274·0	¾	100	200·0	300·0	399·8
2865·0	2°	100	200·0	299·9	399·7
2547·0	¼	100	200·0	299·9	399·6
2292·0	½	100	200·0	299·8	399·5
2084·0	¾	100	200·0	299·8	399·4
1910·0	3°	100	200·0	299·7	399·3
1763·0	¼	100	200·0	299·7	399·2
1637·0	½	100	200·0	299·6	399·1
1528·0	¾	100	200·0	299·6	399·0
1433·0	4°	100	199·9	299·6	398·9
1348·0	¼	100	199·9	299·5	398·7
1274·0	½	100	199·9	299·4	398·5
1207·0	¾	100	199·9	299·3	398·3
1146·0	5°	100	199·9	299·2	398·0
1092·0	¼	100	199·8	299·1	397·8
1042·0	½	100	199·8	299·0	397·6
996·8	¾	100	199·7	298·9	397·5
955.4	6°	100	199·7	298·8	397·3
917·0	¼	100	199·7	298·7	397·0
882·0	½	100	199·7	298·6	396·7
849·3	¾	100	19[illegible]	298·5	396·5
819·0	7°	100	199·6	298·4	396·2
790·8	¼	100	199·6	298·3	396·0
764·5	½	100	199·6	298·2	395·7
739·9	¾	100	199·6	298·1	395·4
716·8	8°	100	199·6	298·0	395·1
695·1	¼	100	199·5	297·9	394·8
674·6	½	100	199·5	297·8	394·5
655·5	¾	100	199·4	297·7	394·3
637·3	9°	100	199·4	297·5	394·1
620·2	¼	100	199·4	297·4	393·7
603·8	½	100	199·3	297·3	393·2
588·4	¾	100	199·2	297·2	392·8
573·7	10°	100	199·2	297·0	392·4

TABLE OF ORDINATES.

Ordinates 10 feet apart.—Chord 100 feet.

Deflexion Angle in Degrees and Minutes.	Distances of the Ordinates from the end of the 100 feet Chord.				
	50 feet.	40 feet.	30 feet.	20 feet.	10 feet.
	Lengths of Ordinates in feet.				
° ′					
5	·018	·017	·015	·012	·006
10	·036	·035	·031	·023	·013
15	·054	·052	·046	·035	·019
20	·073	·070	·061	·047	·026
25	·091	·087	·076	·058	·032
30	·109	·105	·092	·070	·039
35	·127	·123	·108	·082	·045
40	·145	·140	·123	·093	·052
45	·163	·157	·137	·105	·058
50	·182	·175	·153	·117	·065
55	·200	·192	·168	·128	·071
1°	·218	·209	·183	·140	·078
5	·236	·226	·198	·152	·085
10	·254	·244	·214	·163	·091
15	·273	·261	·229	·175	·098
20	·291	·279	·244	·187	·104
25	·309	·296	·259	·198	·111
30	·327	·314	·275	·210	·117
35	·345	·331	·290	·221	·124
40	·364	·349	·305	·233	·130
45	·382	·366	·321	·245	·137
50	·400	·384	·336	·256	·144
55	·418	·401	·351	·268	·150
2°	·436	·419	·366	·280	·157
5	·454	·436	·382	·291	·163
10	·473	·454	·397	·303	·170
15	·491	·471	·412	·315	·176
20	·509	·489	·428	·326	·183
25	·527	·506	·443	·338	·190
30	·545	·524	·458	·350	·196
35	·564	·541	·474	·361	·203
40	·582	·559	·489	·373	·209
45	·600	·576	·504	·384	·216
50	·618	·594	·519	·396	·222
55	·636	·611	·535	·408	·229
3°	·654	·629	·550	·419	·235
5	·673	·646	·565	·431	·242
10	·691	·664	·581	·443	·249
15	·709	·681	·596	·454	·255
20	·727	·699	·611	·466	·262
25	·745	·716	·627	·478	·268

TABLE OF ORDINATES.—CONTINUED.

Ordinates 10 feet apart.—Chord 100 feet.

Deflexion Angle in Degrees and Minutes.		Distances of the Ordinates from the end of the 100 feet Chord.				
		50 feet.	40 feet.	30 feet.	20 feet.	10 feet.
		Lengths of Ordinates in feet.				
°	′					
3	30	·764	·734	·642	·489	·275
	35	·782	·751	·657	·501	·281
	40	·800	·769	·673	·512	·288
	45	·818	·786	·688	·524	·294
	50	·836	·804	·703	·536	·301
	55	·854	·821	·718	·547	·308
4°		·873	·839	·734	·559	·314
	15	·927	·891	·780	·594	·334
	30	·981	·944	·825	·629	·354
	45	1·036	·996	·871	·664	·373
5°		1·091	1·048	·917	·699	·393
	15	1·146	1·100	·963	·734	·413
	30	1·200	1·153	1·009	·769	·432
	45	1·255	1·205	1·055	·804	·452
6°		1·309	1·258	1·100	·839	·472
	15	1·364	1·310	1·146	·874	·492
	30	1·419	1·362	1·192	·909	·511
	45	1·473	1·415	1·238	·944	·531
7°		1·528	1·467	1·284	·979	·551
	15	1·582	1·520	1·330	1·014	·570
	30	1·637	1·572	1·375	1·048	·590
	45	1·692	1·624	1·421	1·083	·610
8°		1·746	1·677	1·467	1·118	·629
	15	1·801	1·729	1·513	1·153	·649
	30	1·855	1·782	1·559	1·188	·669
	45	1·910	1·834	1·605	1·223	·689
9°		1·965	1·886	1·651	1·258	·708
	15	2·019	1·939	1·696	1·293	·728
	30	2·074	1·991	1·742	1·328	·748
	45	2·128	2·044	1·788	1·363	·767
10°		2·183	2·096	1·834	1·398	·787
	15	2·238	2·148	1·880	1·433	·807
	30	2·292	2·201	1·926	1·468	·827
	45	2·347	2·254	1·972	1·503	·846
11°		2·401	2·306	2·018	1·538	·866
	15	2·456	2·359	2·064	1·574	·886
	30	2·511	2·411	2·110	1·609	·906
	45	2·566	2·464	2·156	1·644	·926
12°		2·620	2·516	2·203	1·680	·946
	15	2·675	2·569	2·249	1·715	·966
	30	2·730	2·621	2.295	1·750	·985

TABLE OF ORDINATES.—Continued.

Ordinates 10 feet apart.—Chord 100 feet.

Deflexion Angle in Degrees and Minutes.		Distances of the Ordinates from the end of the 100 feet Chord.				
		50 feet.	40 feet.	30 feet.	20 feet.	10 feet.
		Lengths of Ordinates in feet.				
°	′					
12	45	2·785	2·674	2·341	1·785	1·005
13		2·839	2·726	2·387	1·820	1·025
	15	2·894	2·779	2·433	1·855	1·045
	30	2·949	2·832	2·479	1·891	1·065
	45	3·000	2·884	2·525	1·926	1·085
14°		3·058	2·937	2·571	1·961	1·105
	15	3·113	2·989	2·618	1·996	1·124
	30	3·168	3·042	2·664	2·031	1·144
	45	3·222	3·094	2·710	2·067	1·164
15°		3·277	3·147	2·756	2·102	1·184
	15	3·332	3·200	2·802	2·137	1·204
	30	3·387	3·252	2·848	2·172	1·224
	45	3·442	3·305	2·895	2·208	1·244
16°		3·496	3·358	2·941	2·243	1·264
	30	3·606	3·463	3·033	2·314	1·304
17°		3·716	3·569	3·125	2·384	1·344
	30	3·826	3·674	3·218	2·455	1·384
18°		3·935	3·779	3·310	2·525	1·424
	30	4·045	3·885	3·403	2·596	1·464
19°		4·155	3·990	3·495	2·666	1·504
	30	4·265	4·096	3·588	2·737	1·544
20°		4·375	4·201	3·680	2·808	1·583

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
1	1	1·0000	44	1936	6·6332
2	4	1·4142	45	2025	6·7082
3	9	1·7320	46	2116	6·7823
4	16	2·0000	47	2209	6·8556
5	25	2·2360	48	2304	6·9282
6	36	2·4495	49	2401	7·0000
7	49	2·6457	50	2500	7·0711
8	64	2·8284	51	2601	7·1414
9	81	3·0000	52	2704	7·2111
10	100	3·1623	53	2809	7·2801
11	121	3·3166	54	2916	7·3485
12	144	3·4641	55	3025	7·4162
13	169	3·6055	56	3136	7·4833
14	196	3·7416	57	3249	7·5498
15	225	3·8730	58	3364	7·6158
16	256	4·0000	59	3481	7·6811
17	289	4·1231	60	3600	7·7460
18	324	4·2426	61	3721	7·8102
19	361	4·3589	62	3844	7·8740
20	400	4·4721	63	3969	7·9372
21	441	4·5826	64	4096	8·0000
22	484	4·6904	65	4225	8·0623
23	529	4·7958	66	4356	8·1240
24	576	4·8990	67	4489	8·1853
25	625	5·0000	68	4624	8·2462
26	676	5·0990	69	4761	8·3066
27	729	5·1961	70	4900	8·3666
28	784	5·2915	71	5041	8·4261
29	841	5·3852	72	5184	8·4853
30	900	5·4772	73	5329	8·5440
31	961	5·5678	74	5476	8·6023
32	1024	5·6568	75	5625	8·6603
33	1089	5·7446	76	5776	8·7178
34	1156	5·8309	77	5929	8·7750
35	1225	5·9161	78	6084	8·8318
36	1296	6·0000	79	6241	8·8882
37	1369	6·0828	80	6400	8·9443
38	1444	6·1644	81	6561	9·0000
39	1521	6·2450	82	6724	9·0554
40	1600	6·3246	83	6889	9·1104
41	1681	6·4031	84	7056	9·1651
42	1764	6·4807	85	7225	9·2195
43	1849	6·5574	86	7396	9·2736

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
87	7569	9·3274	130	16900	11·4017
88	7744	9·3808	131	17161	11·4455
89	7921	9·4340	132	17424	11·4891
90	8100	9·4868	133	17689	11·5326
91	8281	9·5394	134	17956	11·5758
92	8464	9·5917	135	18225	11·6189
93	8649	9·6436	136	18496	11·6619
94	8836	9·6954	137	18769	11·7047
95	9025	9·7468	138	19044	11·7473
96	9216	9·7979	139	19321	11·7898
97	9409	9·8488	140	19600	11·8322
98	9604	9·8995	141	19881	11·8743
99	9801	9·9499	142	20164	11·9164
100	10000	10·0000	143	20449	11·9583
101	10201	10·0499	144	20736	12·0000
102	10404	10·0995	145	21025	12·0416
103	10609	10·1489	146	21316	12·0830
104	10816	10·1980	147	21609	12·1244
105	11025	10·2469	148	21904	12·1655
106	11236	10·2956	149	22201	12·2065
107	11449	10·3440	150	22500	12·2474
108	11664	10·3923	151	22801	12·2882
109	11881	10·4403	152	23104	12·3288
110	12100	10·4881	153	23409	12·3693
111	12321	10·5356	154	23716	12·4097
112	12544	10·5830	155	24025	12·4499
113	12769	10·6301	156	24336	12·4900
114	12996	10·6771	157	24649	12·5300
115	13225	10·7238	158	24964	12·5700
116	13456	10·7703	159	25281	12·6095
117	13689	10·8166	160	25600	12·6491
118	13924	10·8628	161	25921	12·6886
119	14161	10·9087	162	26244	12·7279
120	14400	10·9544	163	26569	12·7671
121	14641	11·0000	164	26896	12·8062
122	14884	11·0454	165	27225	12·8452
123	15129	11·0905	166	27556	12·8841
124	15376	11·1355	167	27889	12·9228
125	15625	11·1803	168	28224	12·9615
126	15876	11.2250	169	28561	13·0000
127	16129	11·2694	170	28900	13·0384
128	16384	11·3137	171	29241	13·0767
129	16641	11·3578	172	29584	13·1149

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—CONTINUED.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
173	29929	13·1529	216	46656	14·6969
174	30276	13·1909	217	47089	14·7309
175	30625	13·2287	218	47524	14·7648
176	30976	13·2665	219	47961	14·7986
177	31329	13·3041	220	48400	14·8324
178	31684	13·3417	221	48841	14·8661
179	32041	13·3791	222	49284	14·8997
180	32400	13·4164	223	49729	14·9332
181	32761	13·4536	224	50176	14·9666
182	33124	13·4907	225	50625	15·0000
183	33489	13·5277	226	51076	15·0333
184	33856	13·5647	227	51529	15·0665
185	34225	13·6015	228	51984	15·0997
186	34596	13·6382	229	52441	15·1327
187	34969	13·6748	230	52900	15·1657
188	35344	13·7113	231	53361	15·1987
189	35721	13·7477	232	53824	15·2315
190	36100	13·7840	233	54289	15·2643
191	36481	13·8203	234	54756	15·2970
192	36864	13·8564	235	55225	15·3297
193	37249	13·8924	236	55696	15·3623
194	37636	13·9284	237	56169	15·3948
195	38025	13·9642	238	56644	15·4272
196	38416	14·0000	239	57121	15·4596
197	38809	14·0357	240	57600	15·4919
198	39204	14·0712	241	58081	15·5242
199	39601	14·1067	242	58564	15·5563
200	40000	14·1421	243	59049	15·5885
201	40401	14·1774	244	59536	15·6205
202	40804	14·2127	245	60025	15·6525
203	41209	14·2478	246	60516	15·6844
204	41616	14·2828	247	61009	15·7162
205	42025	14·3178	248	61504	15·7480
206	42436	14·3527	249	62001	15·7797
207	42849	14·3874	250	62500	15·8114
208	43264	14·4222	251	63001	15·8430
209	43681	14·4568	252	63504	15·8745
210	44100	14·4914	253	64009	15·9060
211	44521	14·5258	254	64516	15·9374
212	44944	14·5602	255	65025	15·9687
213	45369	14·5945	256	65536	16·0000
214	45796	14·6287	257	66049	16·0312
215	46225	14·6629	258	66564	16·0624

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
259	67081	16.0935	302	91204	17·3781
260	67600	16·1245	303	91809	17·4069
261	68121	16·1555	304	92416	17·4356
262	68644	16·1864	305	93025	17·4642
263	69169	16·2173	306	93636	17·4928
264	69696	16·2481	307	94249	17·5214
265	70225	16·2788	308	94864	17·5499
266	70756	16·3095	309	95481	17·5784
267	71289	16·3401	310	96100	17·6068
268	71824	16·3707	311	96721	17·6352
269	72361	16·4012	312	97344	17·6635
270	72900	16·4317	313	97969	17·6918
271	73441	16·4621	314	98596	17·7200
272	73984	16·4924	315	99225	17·7482
273	74529	16·5227	316	99856	17·7764
274	75076	16·5529	317	100489	17·8045
275	75625	16·5831	318	101124	17·8325
276	76176	16·6132	319	101761	17·8606
277	76729	16·6433	320	102400	17·8885
278	77284	16·6733	321	103041	17·9165
279	77841	16·7033	322	103684	17·9444
280	78400	16·7332	323	104329	17·9722
281	78961	16·7630	324	104976	18·0000
282	79524	16·7928	325	105625	18·0277
283	80089	16·8226	326	106276	18·0555
284	80656	16·8523	327	106929	18·0831
285	81225	16·8819	328	107584	18·1108
286	81796	16·9115	329	108241	18·1384
287	82369	16·9411	330	108900	18·1659
288	82944	16·9706	331	109561	18·1934
289	83521	17·0000	332	110224	18·2209
290	84100	17·0294	333	110889	18·2483
291	84681	17·0587	334	111556	18·2757
292	85264	17·0880	335	112225	18·3030
293	85849	17·1172	336	112896	18·3303
294	86436	17·1464	337	113569	18·3576
295	87025	17·1756	338	114244	18·3848
296	87616	17·2046	339	114921	18·4119
297	88209	17·2337	340	115600	18·4391
298	88804	17·2627	341	116281	18·4662
299	89401	17·2916	342	116964	18·4932
300	90000	17·3205	343	117649	18·5203
301	90601	17·3493	344	118336	18·5472

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—CONTINUED.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
345	119025	18·5742	388	150544	19·6977
346	119716	18·6011	389	151321	19·7231
347	120409	18·6279	390	152100	19·7484
348	121104	18·6548	391	152881	19·7737
349	121801	18·6815	392	153664	19·7990
350	122500	18·7083	393	154449	19·8242
351	123201	18·7350	394	155236	19·8494
352	123904	18·7617	395	156025	19·8746
353	124609	18·7883	396	156816	19·8997
354	125316	18·8149	397	157609	19·9248
355	126025	18·8414	398	158404	19·9499
356	126736	18·8680	399	159201	19·9750
357	127449	18·8944	400	160000	20·0000
358	128164	18·9209	401	160801	20·0250
359	128881	18·9473	402	161604	20·0499
360	129600	18·9737	403	162409	20·0749
361	130321	19·0000	404	163216	20·0997
362	131044	19·0263	405	164025	20·1246
363	131769	19·0526	406	164836	20·1494
364	132496	19·0788	407	165649	20·1742
365	133225	19·1050	408	166464	20·1990
366	133956	19·1311	409	167281	20·2237
367	134689	19·1572	410	168100	20·2485
368	135424	19·1833	411	168921	20·2731
369	136161	19·2094	412	169744	20·2978
370	136900	19·2354	413	170569	20·3224
371	137641	19·2614	414	171396	20·3470
372	138384	19·2873	415	172225	20·3715
373	139129	19·3132	416	173056	20·3961
374	139876	19·3391	417	173889	20·4206
375	140625	19·3649	418	174724	20·4450
376	141376	19·3907	419	175561	20·4695
377	142129	19·4165	420	176400	20·4939
378	142884	19·4422	421	177241	20·5183
379	143641	19·4679	422	178084	20·5426
380	144400	19·4936	423	178929	20·5670
381	145161	19·5192	424	179776	20·5913
382	145924	19·5448	425	180625	20·6155
383	146689	19·5704	426	181476	20·6398
384	147456	19·5959	427	182329	20·6640
385	148225	19·6214	428	183184	20·6882
386	148996	19·6469	429	184041	20·7123
387	149769	19·6723	430	184900	20·7364

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
431	185761	20·7605	474	224676	21·7715
432	186624	20·7846	475	225625	21·7945
433	187489	20·8086	476	226576	21·8174
434	188356	20·8327	477	227529	21·8403
435	189225	20·8566	478	228484	21·8632
436	190096	20·8806	479	229441	21·8861
437	190969	20·9045	480	230400	21·9089
438	191844	20·9284	481	231361	21·9317
439	192721	20·9523	482	232324	21·9545
440	193600	20·9762	483	233289	21·9773
441	194481	21·0000	484	234256	22·0000
442	195364	21·0238	485	235225	22·0227
443	196249	21·0476	486	236196	22·0454
444	197136	21·0713	487	237169	22·0689
445	198025	21·0950	488	238144	22·0907
446	198916	21·1187	489	239121	22·1133
447	199809	21·1424	490	240100	22·1359
448	200704	21·1660	491	241081	22·1585
449	201601	21·1896	492	242064	22·1811
450	202500	21·2132	493	243049	22·2036
451	203401	21·2368	494	244036	22·2261
452	204304	21·2603	495	245025	22·2486
453	205209	21·2838	496	246016	22·2711
454	206116	21·3073	497	247009	22·2935
455	207025	21·3307	498	248004	22·3159
456	207936	21·3542	499	249001	22·3383
457	208849	21·3776	500	250000	22·3607
458	209764	21·4009	501	251001	22·3830
459	210681	21·4243	502	252004	22·4054
460	211600	21·4476	503	253009	22·4277
461	212521	21·4709	504	254016	22·4499
462	213444	21·4942	505	255025	22·4722
463	214369	21·5174	506	256036	22·4944
464	215296	21·5407	507	257049	22·5167
465	216225	21·5639	508	258064	22·5388
466	217156	21·5870	509	259081	22·5610
467	218089	21·6102	510	260100	22·5832
468	219024	21·6333	511	261121	22·6053
469	219961	21·6564	512	262144	22·6274
470	220900	21·6795	513	263169	22·6495
471	221841	21·7025	514	264196	22·6716
472	222784	21·7256	515	265225	22·6936
473	223729	21·7486	516	266256	22·7156

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
517	267289	22·7376	560	313600	23·6643
518	268324	22·7596	561	314721	23·6854
519	269361	22·7816	562	315844	23·7065
520	270400	22·8035	563	316969	23·7276
521	271441	22·8254	564	318096	23·7487
522	272484	22·8473	565	319225	23·7697
523	273529	22·8692	566	320356	23·7907
524	274576	22·8910	567	321489	23·8118
525	275625	22·9129	568	322624	23·8327
526	276676	22·9347	569	323761	23·8537
527	277729	22·9565	570	324900	23·8747
528	278784	22·9782	571	326041	23·8956
529	279841	23·0000	572	327184	23·9165
530	280900	23·0217	573	328329	23·9374
531	281961	23·0434	574	329476	23·9583
532	283024	23·0651	575	330625	23·9792
533	284089	23·0868	576	331776	24·0000
534	285156	23·1084	577	332929	24·0208
535	286225	23·1301	578	334084	24·0416
536	287296	23·1517	579	335241	24·0624
537	288369	23·1733	580	336400	24·0832
538	289444	23·1948	581	337561	24·1039
539	290521	23·2164	582	338724	24·1247
540	291600	23·2379	583	339889	24·1454
541	292681	23·2594	584	341056	24·1661
542	293764	23·2809	585	342225	24·1868
543	294849	23·3021	586	343396	24·2074
544	295936	23·3238	587	344569	24·2281
545	297025	23·3452	588	345744	24·2487
546	298116	23·3666	589	346921	24·2693
547	299209	23·3880	590	348100	24·2899
548	300304	23·4094	591	349281	24·3105
549	301401	23·4307	592	350464	24·3310
550	302500	23·4521	593	351649	24·3516
551	303601	23·4734	594	352836	24·3721
552	304704	23·4947	595	354025	24·3926
553	305809	23·5159	596	355216	24·4131
554	306916	23·5372	597	356409	24·4336
555	308025	23·5584	598	357604	24·4540
556	309136	23·5796	599	358801	24·4745
557	310249	23·6008	600	360000	24·4949
558	311364	23·6220	601	361201	24·5153
559	312481	23·6432	602	362404	24·5357

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
603	363609	24·5560	646	417316	25·4165
604	364816	24·5764	647	418609	25·4362
605	366025	24·5967	648	419904	25·4558
606	367236	24·6171	649	421201	25·4755
607	368449	24·6374	650	422500	25·4950
608	369664	24·6576	651	423801	25·5147
609	370881	24·6779	652	425104	25·5343
610	372100	24·6982	653	426409	25·5539
611	373321	24·7184	654	427716	25·5734
612	374544	24·7386	655	429025	25·5930
613	375769	24·7588	656	430336	25·6125
614	376996	24·7790	657	431649	25·6320
615	378225	24·7992	658	432964	25·6515
616	379456	24·8193	659	434281	25·6710
617	380689	24·8395	660	435600	25·6905
618	381924	24·8596	661	436921	25·7099
619	383161	24·8797	662	438244	25·7204
620	384400	24·8998	663	439569	25·7488
621	385641	24·9199	664	440896	25·7682
622	386884	24·9399	665	442225	25·7876
623	388129	24·9600	666	443556	25·8070
624	389376	24·9800	667	444889	25·8263
625	390625	25·0000	668	446224	25·8457
626	391876	25·0200	669	447561	25·8650
627	393129	25·0400	670	448900	25·8844
628	394384	25·0600	671	450241	25·9037
629	395641	25·0799	672	451584	25·9230
630	396900	25·0998	673	452929	25·9422
631	398161	25·1197	674	454276	25·9615
632	399424	25·1396	675	455625	25·9808
633	400689	25·1595	676	456976	26·0000
634	401956	25·1794	677	458329	26·0192
635	403225	25·1992	678	459684	26·0384
636	404496	25·2190	679	461041	26·0576
637	405769	25·2389	680	462400	26·0768
638	407044	25·2587	681	463761	26·0960
639	408321	25·2785	682	465124	26·1151
640	409600	25·2982	683	466489	26·1343
641	410881	25·3180	684	467856	26·1534
642	412164	25·3377	685	469225	26·1725
643	413449	25·3574	686	470596	26·1916
644	414736	25·3772	687	471969	26·2107
645	416025	25·3968	688	473344	26·2297

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
689	474721	26·2488	732	535824	27·0555
690	476100	26·2678	733	537289	27·0740
691	477481	26·2869	734	538756	27·0924
692	478864	26·3059	735	540225	27·1109
693	480249	26·3249	736	541696	27·1293
694	481636	26·3439	737	543169	27·1477
695	483025	26·3629	738	544644	27·1662
696	484416	26·3818	739	546121	27·1846
697	485809	26·4008	740	547600	27·2029
698	487204	26·4197	741	549081	27·2213
699	488601	26·4386	742	550564	27·2397
700	490000	26·4575	743	552049	27·2580
701	491401	26·4764	744	553536	27·2764
702	492804	26·4953	745	555025	27·2947
703	494209	26·5141	746	556516	27·3130
704	495616	26·5330	747	558009	27·3313
705	497025	26·5518	748	559504	27·3496
706	498436	26·5707	749	561001	27·3679
707	499849	26·5895	750	562500	27·3861
708	501264	26·6083	751	564001	27·4044
709	502681	26·6271	752	565504	27·4226
710	504100	26·6458	753	567009	27·4408
711	505521	26·6646	754	568516	27·4591
712	506944	26·6833	755	570025	27·4773
713	508369	26·7021	756	571536	27·4955
714	509796	26·7208	757	573049	27·5136
715	511225	26·7395	758	574564	27·5318
716	512656	26·7582	759	576081	27·5500
717	514089	26·7769	760	577600	27·5681
718	515524	26·7955	761	579121	27·5862
719	516961	26·8142	762	580644	27·6043
720	518400	26·8328	763	582169	27·6225
721	519841	26·8514	764	583696	27·6405
722	521284	26·8701	765	585225	27·6586
723	522729	26·8887	766	586756	27·6767
724	524176	26·9072	767	588289	27·6948
725	525625	26·9258	768	589824	27·7128
726	527076	26·9444	769	591361	27·7308
727	528529	26·9629	770	592900	27·7489
728	529984	26·9815	771	594441	27·7669
729	531441	27·0000	772	595984	27·7849
730	532900	27·0185	773	597529	27·8029
731	534361	27·0370	774	599076	27·8209

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—CONTINUED.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
775	600625	27·8388	818	669124	28·6007
776	602176	27·8568	819	670761	28·6182
777	603729	27·8747	820	672400	28·6356
778	605284	27·8926	821	674041	28·6531
779	606841	27·9106	822	675684	28·6705
780	608400	27·9285	823	677329	28·6880
781	609961	27·9464	824	678976	28·7054
782	611524	27·9643	825	680625	28·7228
783	613089	27·9821	826	682276	28·7402
784	614656	28·0000	827	683929	28·7576
785	616225	28·0179	828	685584	28·7750
786	617796	28·0357	829	687241	28·7924
787	619369	28·0535	830	688900	28·8097
788	620944	28·0713	831	690561	28·8271
789	622521	28·0891	832	692224	28·8444
790	624100	28·1069	833	693889	28·8617
791	625681	28·1247	834	695556	28·8791
792	627264	28·1425	835	697225	28·8964
793	628849	28·1603	836	698896	28·9137
794	630436	28·1780	837	700569	28·9310
795	632025	28·1957	838	702244	28·9482
796	633616	28·2135	839	703921	28·9655
797	635209	28·2312	840	705600	28·9828
798	636804	28·2489	841	707281	29·0000
799	638401	28·2666	842	708964	29·0172
800	640000	28·2843	843	710649	29·0345
801	641601	28·3019	844	712336	29·0517
802	643204	28·3196	845	714025	29·0689
803	644809	28·3373	846	715716	29·0861
804	646416	28·3549	847	717409	29·1033
805	648025	28·3726	848	719104	29·1204
806	649636	28·3901	849	720801	29·1376
807	651249	28·4077	850	722500	29·1548
808	652864	28·4253	851	724201	29·1719
809	654481	28·4429	852	725904	29·1890
810	656100	28·4605	853	727609	29·2062
811	657721	28·4781	854	729316	29·2233
812	659344	28·4956	855	731025	29·2404
813	660969	28·5132	856	732736	29·2575
814	662596	28·5307	857	734449	29·2746
815	664225	28·5482	858	736164	29·2916
816	665856	28·5657	859	737881	29·3087
817	667489	28·5832	860	739600	29·3258

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
861	741321	29·3428	904	817216	30·0666
862	743044	29·3598	905	819025	30·0832
863	744769	29·3769	906	820836	30·0998
864	746496	29·3939	907	822649	30·1164
865	748225	29·4109	908	824464	30·1330
866	749956	29·4279	909	826281	30·1496
867	751689	29·4449	910	828100	30·1662
868	753424	29·4618	911	829921	30·1828
869	755161	29·4788	912	831744	30·1993
870	756900	29·4958	913	833569	30 2159
871	758641	29·5127	914	835396	30·2324
872	760384	29·5296	915	837225	30·2490
873	762129	29·5466	916	839056	30·2655
874	763876	29·5635	917	840889	30·2820
875	765625	29·5804	918	842724	30·2985
876	767376	29·5973	919	844561	30·3150
877	769129	29·6142	920	846400	30·3315
878	770884	29·6311	921	848241	30·3480
879	772641	29·6479	922	850084	30·3645
880	774400	29·6648	923	851929	30·3809
881	776161	29·6816	924	853776	30·3974
882	777924	29·6985	925	855625	30·4138
883	779689	29·7153	926	857476	30·4302
884	781456	29·7321	927	859329	30·4467
885	783225	29·7489	928	861184	30·4631
886	784996	29·7658	929	863041	30·4795
887	786769	29·7825	930	864900	30·4959
888	788544	29·7993	931	866761	30·5123
889	790321	29·8161	932	868624	30·5287
890	792100	29·8329	933	870489	30·5450
891	793881	29·8496	934	872356	30·5614
892	795664	29·8664	935	874225	30·5778
893	797449	29·8831	936	876096	30·5941
894	799236	29·8998	937	877969	30·6105
895	801025	29·9166	938	879844	30·6268
896	802816	29·9333	939	881721	30·6431
897	804609	29·9500	940	883600	30·6594
898	806404	29·9666	941	885481	30·6757
899	808201	29·9833	942	887364	30·6920
900	810000	30·0000	943	889249	30·7083
901	811801	30·0167	944	891136	30·7246
902	813604	30·0333	945	893025	30·7409
903	815409	30·0500	946	894916	30·7571

A TABLE OF THE SQUARES AND SQUARE ROOTS OF NUMBERS.—Continued.

From 1 to 1000.

No.	Squares.	Square Roots.	No.	Squares.	Square Roots.
947	896809	30·7734	974	948676	31·2090
948	898704	30·7896	975	950625	31·2250
949	900601	30·8058	976	952576	31·2410
950	902500	30·8221	977	954529	31·2570
951	904401	30·8383	978	956484	31·2730
952	906304	30·8545	979	958441	31·2890
953	908209	30·8707	980	960400	31·3050
954	910116	30·8869	981	962361	31·3209
955	912025	30·9031	982	964324	31·3369
956	913936	30·9192	983	966289	31·3528
957	915849	30·9354	984	968256	31·3688
958	917764	30·9516	985	970225	31·3847
959	919681	30·9677	986	972196	31·4006
960	921600	30·9839	987	974169	31·4166
961	923521	31·0000	988	976144	31·4325
962	925444	31·0161	989	978121	31·4484
963	927369	31·0322	990	980100	31·4643
964	929296	31·0483	991	982081	31·4802
965	931225	31·0644	992	984064	31·4960
966	933156	31·0805	993	986049	31·5119
967	935089	31·0966	994	988036	31·5278
968	937024	31·1127	995	990025	31·5436
969	938961	31·1288	996	992016	31·5595
970	940900	31·1448	997	994009	31·5753
971	942841	31·1609	998	996004	31·5911
972	944784	31·1769	999	998001	31·6070
973	946729	31·1929	1000	1000000	31·6228

TABLE OF SLOPES, &c.—For Topography.

Degrees.	Vertical Rise in 100 feet horizontal.	Horizontal Distance to a rise of 10 feet.	Degrees.	Vertical Rise in 100 feet horizontal.	Horizontal Distance to a rise of 10 feet.
°			°		
1	1·75	572·9	19	34·43	29·0
2	3·49	286·4	20	36·40	27·5
3	5·24	190·8	21	38·40	26·0
4	6·99	143·0	22	40·40	24·7
5	8·75	114·3	23	42·45	23·5
6	10·51	95·1	24	44·52	22·4
7	12·28	81·4	25	46·63	21·4
8	14·05	71·2	26	48·77	20·5
9	15·83	63·1	27	50·95	19·6
10	17·63	56·7	28	53·17	18·8
11	19·44	51·4	29	55·43	18·0
12	21·25	47·0	30	57·73	17·3
13	23·09	43·3	35	70·02	14·2
14	24·93	40·1	40	83·91	11·9
15	26·79	37·3	45	100·00	10·0
16	28·67	34·9	50	119·17	8·4
17	30·57	32·7	55	142·81	7·0
18	32·49	30·7	60	173·20	5·7

E. B. MEARS, STEREOTYPER.